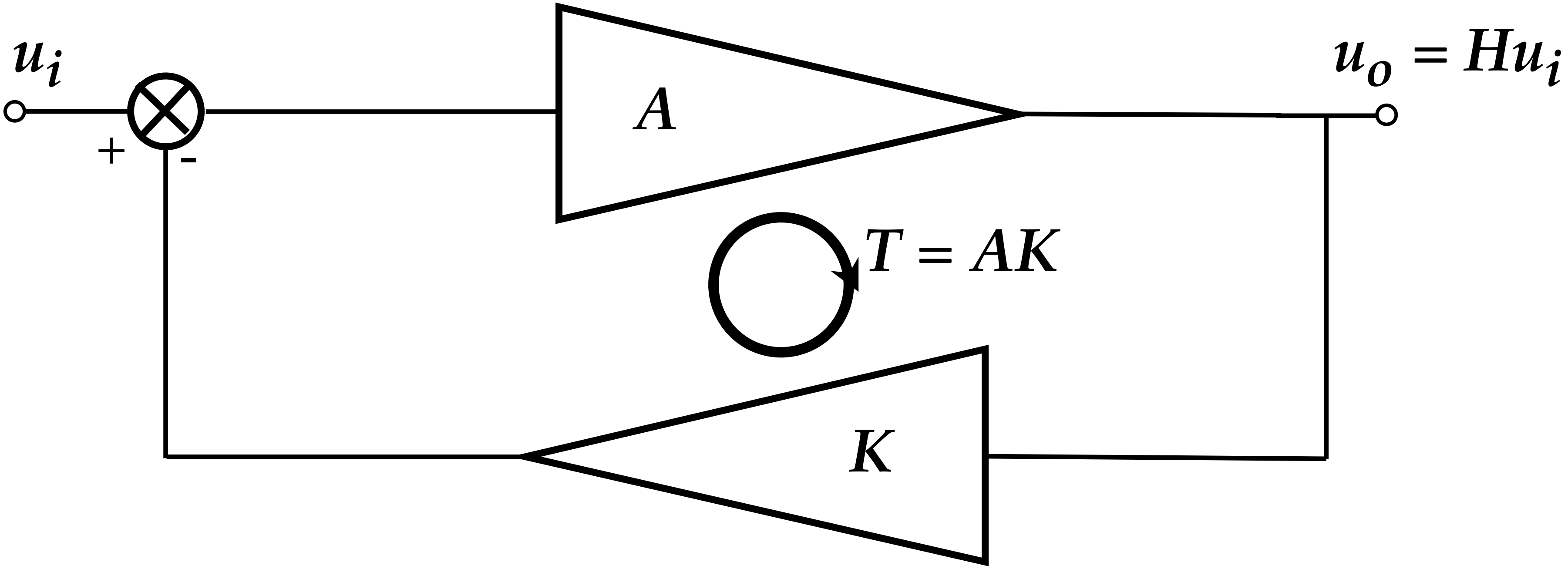
11. NDI AND THE GFT:

Null Double Injection and the General Feedback Theorem

How to identify and include nonidealities in a third quantity, the Null Loop Gain T_n

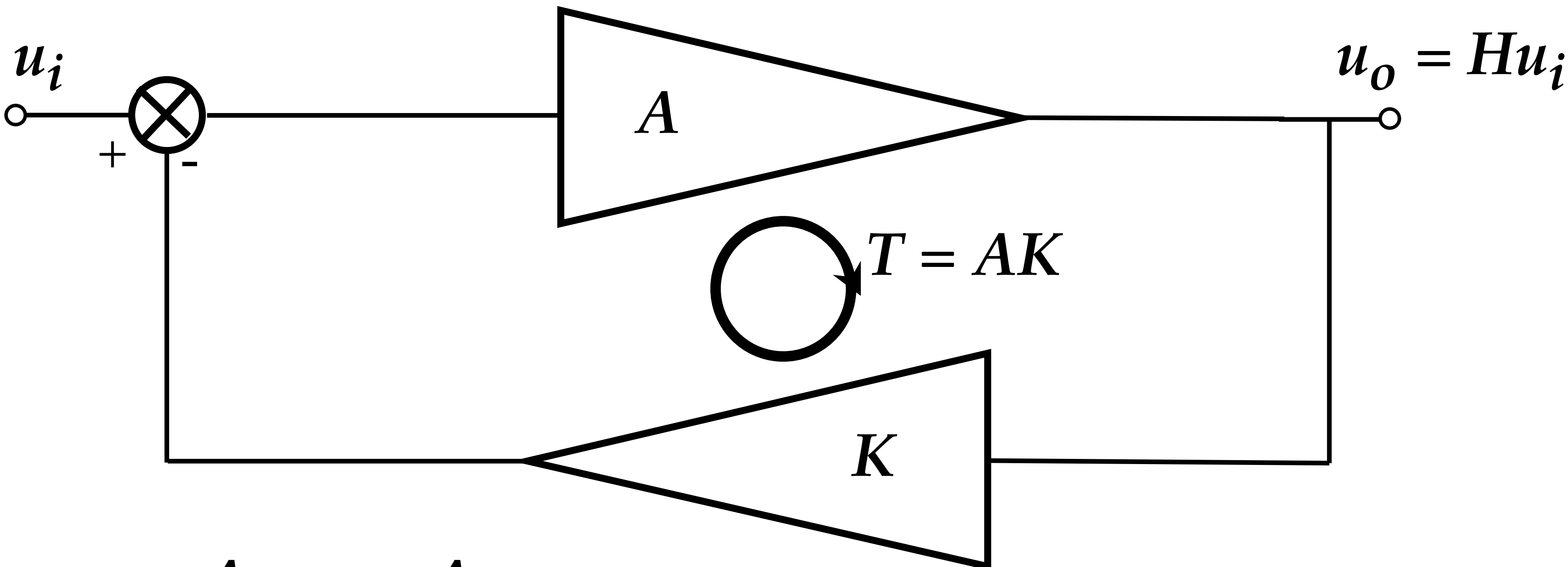
Conventional block diagram:



$$H = \frac{A}{1 + AK} = \frac{A}{1 + T}$$

where $T \equiv AK$ is the loop gain

Block diagram approach for closed-loop gain H:

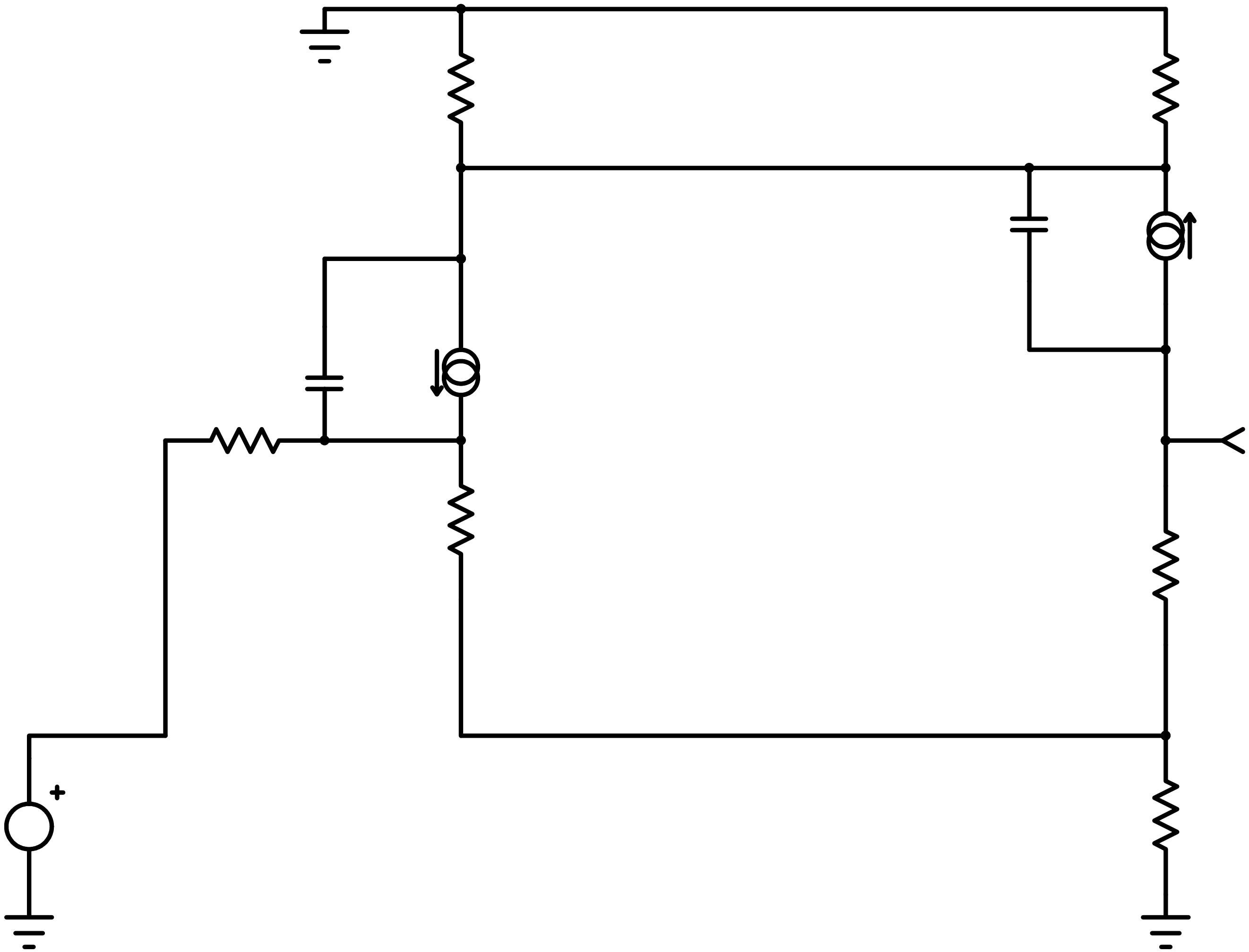


$$H = \frac{A}{1 + AK} = \frac{A}{1 + T}$$

where $T \equiv AK$ is the loop gain

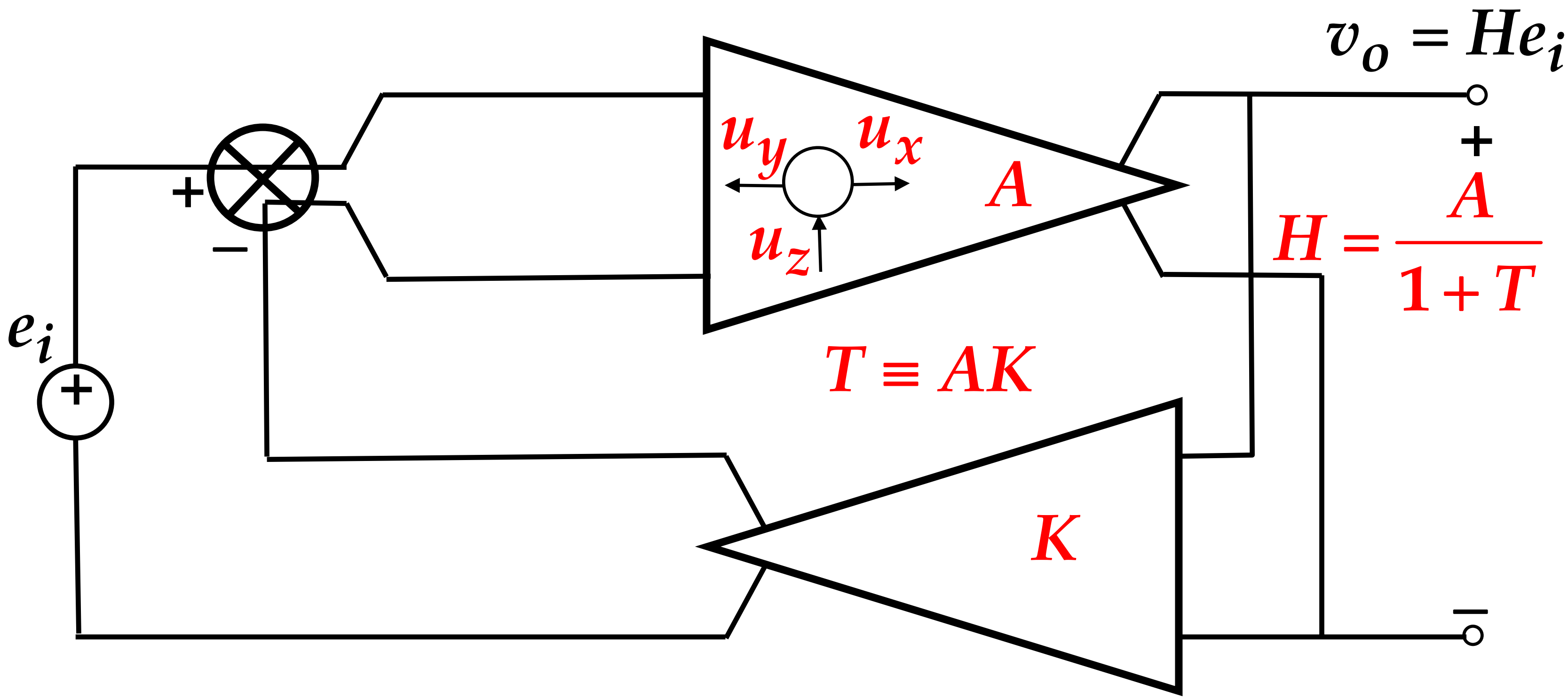
The block diagram says: if $A = 0$, then $H = 0$

But, this isn't true in an actual circuit model:



If the second device fails open, the input signal can still reach the output by going the "wrong way" through the feedback path.

The formula is wrong because it is based on an incomplete model:



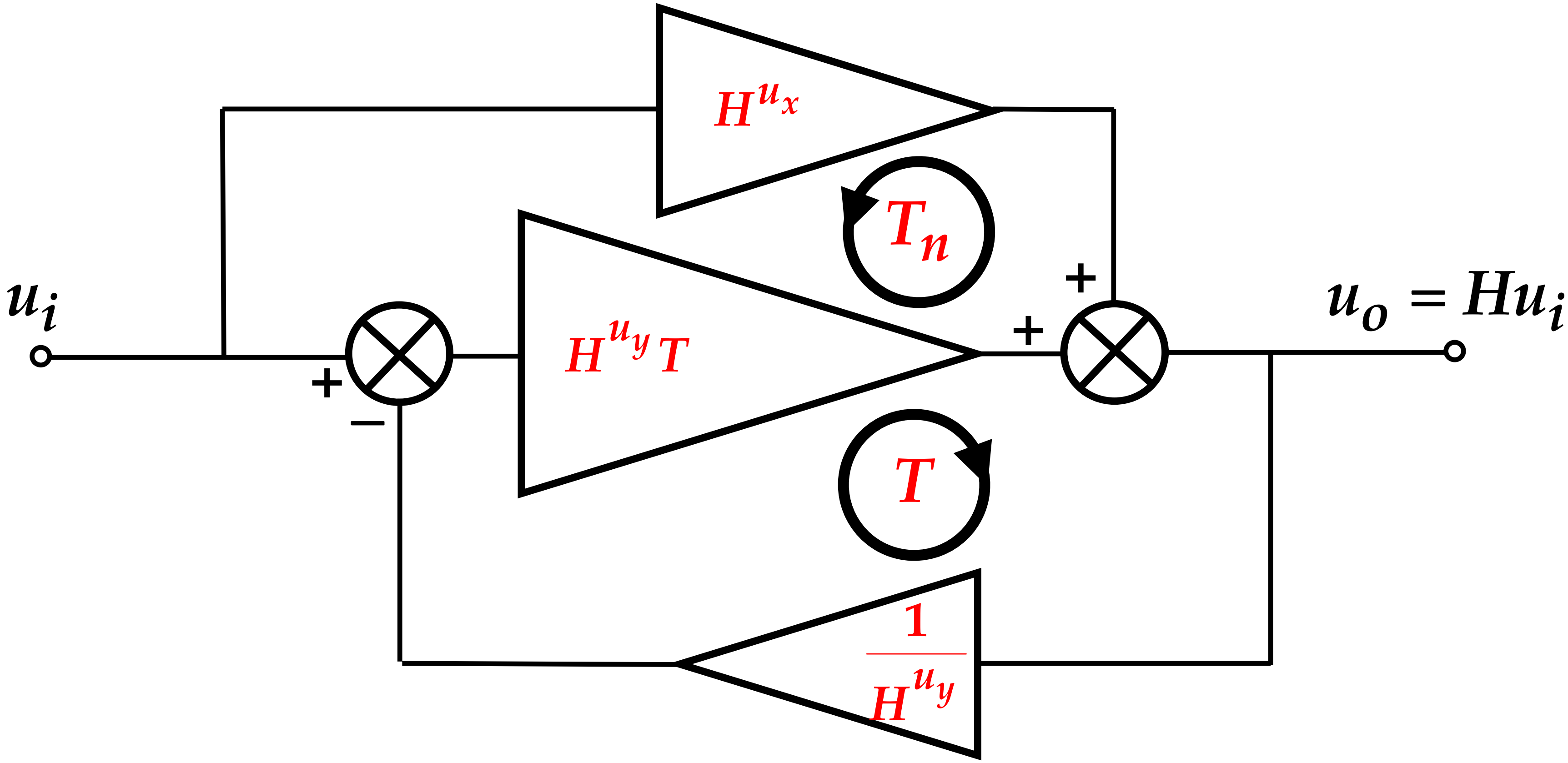
Two deficiencies:

1. Requires an ideal injection point

2. Ignores nonidealities

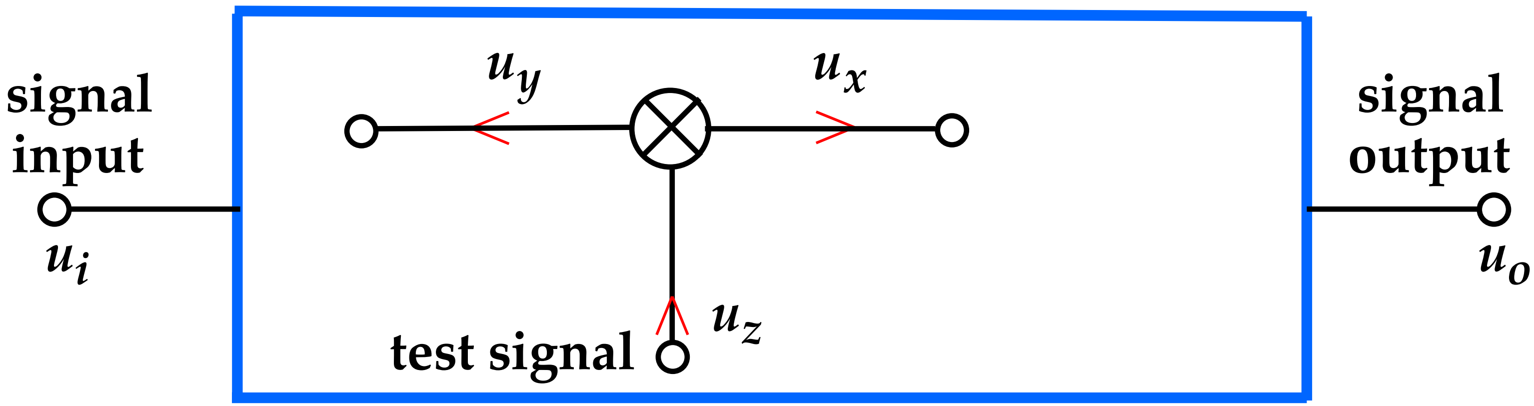
In contrast, the Dissection Theorem, which is a formula in similar format, is not based on a model.

On the contrary, the block diagram is a *result* of the formula, not its *origin*, and contains no assumptions or approximations.



How can the DT be made to represent a feedback system?

Dissection Theorem (DT)



$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0}$$

$$H = H^{u_y}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}$$

ndi ndi ndi si

first level TF

second level TFs

$$H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

$$H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0}$$

Redundancy Relation:

$$\frac{H^{u_y}}{H^{u_x}} = \frac{T_n}{T}$$

There are many reasons why the Dissection Theorem is useful.

The *minimum* benefit of the DT is that it embodies the "Divide and Conquer" approach, because one complicated calculation is replaced by three calculations, two of which are ndi calculations and are therefore *simpler* and *easier* than si calculations.

Not only does the DT implement the Design & Conquer objective, but the DT is itself a Low Entropy Expression, and *much greater* benefits accrue if the second level TFs have useful physical interpretations.

Thus, the second level TFs themselves contain the useful design-oriented information and you may never need to actually substitute them into the theorem.

For example, if $T, T_n >> 1$, $H \approx H^{u_y}$

How to determine the physical interpretations of the second level TFs?

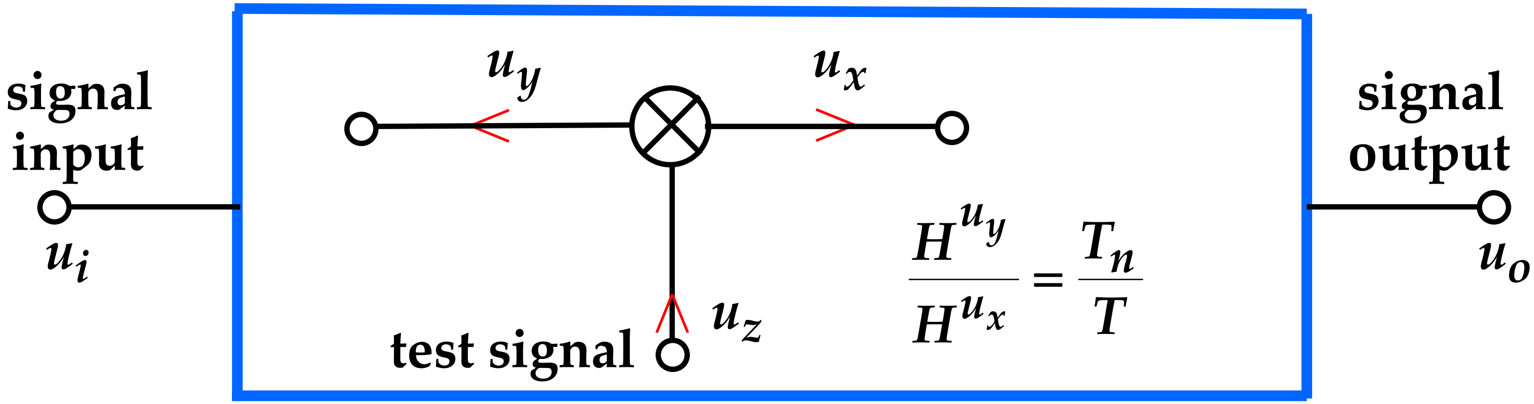
What kind of signal (voltage or current) is injected, and where it is injected, defines an "injection configuration."

Therefore, ***the key decision in applying the DT is choosing a test signal injection point so that at least one of the second level TFs has the physical interpretation you want it to have.***

Specific injection configurations for the DT lead to the:

- **Extra Element Theorem (EET)**
- **Chain Theorem (CT)**
- **General Feedback Theorem (GFT)**

Dissection Theorem (DT)



ndi ndi ndi

$$H = H^{u_y}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}$$

si

first level TF — second level TFs

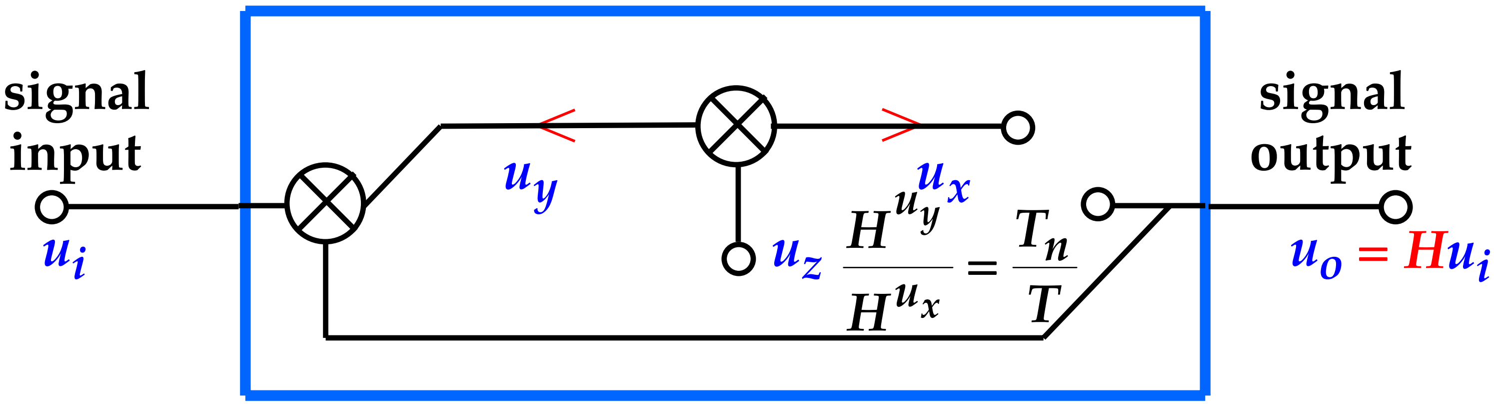
$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \quad H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \quad H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \quad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \quad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

gain

What test signal injection configuration makes the DT represent a feedback system?

We want H^{u_y} to represent the ideal closed-loop gain, and so the test signal u_z must be injected at the error signal summing point.
At the same time, u_x is the signal going forward around the feedback loop, so T represents the loop gain:

Test signal injection at the error signal summing point:



ndi ndi ndi

$$H = H^{u_y} \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y}\frac{T}{1+T} + H^{u_x}\frac{1}{1+T}$$

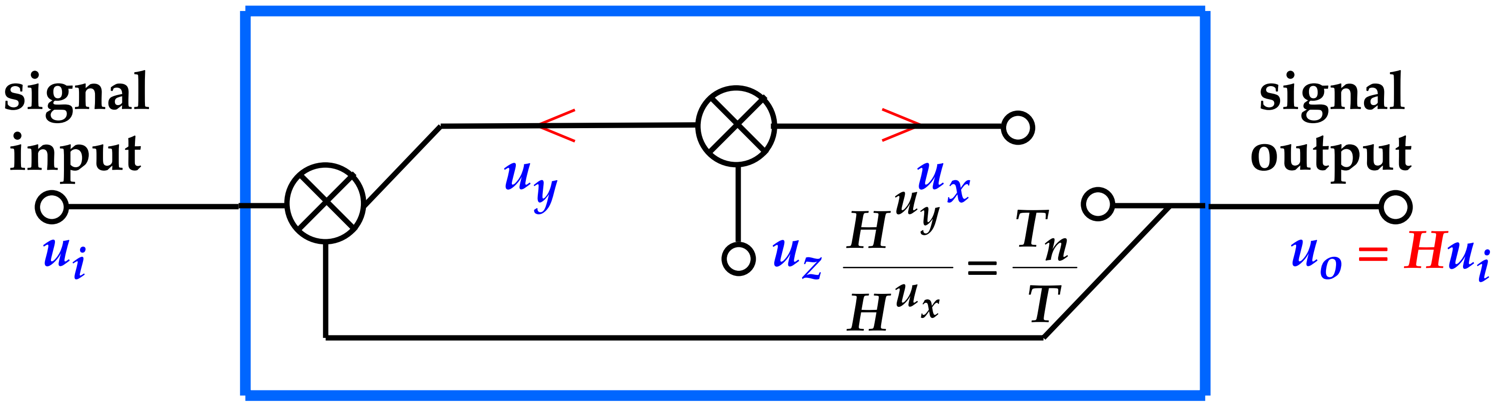
si

first level TF — second level TFs

$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H^{u_y} \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad H^{u_x} \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

closed-loop gain — ideal closed-loop gain — loop gain — null loop gain

Test signal injection at the error signal summing point:



$$H = H^{u_y} \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H^{u_y} \frac{T}{1+T} + H^{u_x} \frac{1}{1+T}$$

It is seen that the closed-loop gain H is the weighted sum of two components:

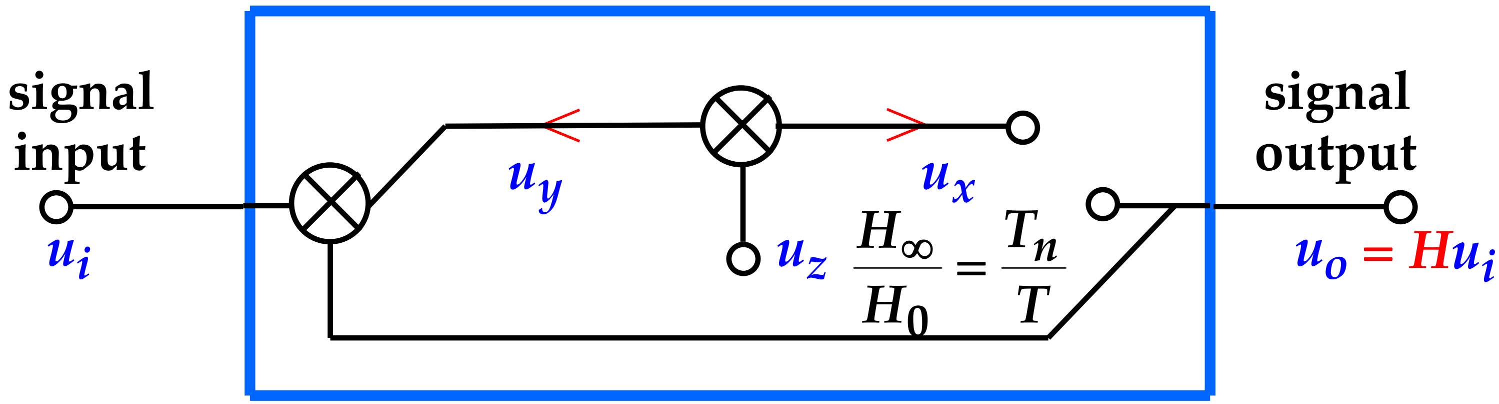
closed-loop gain when $T = \infty$: $H_\infty \equiv H^{u_y}$

(the "ideal closed-loop gain")

closed-loop gain when $T = 0$: $H_0 \equiv H^{u_x}$

(the "direct forward transmission")

With these new definitions, the DT morphs into the
General Feedback Theorem (GFT):



ndi ndi ndi

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

si

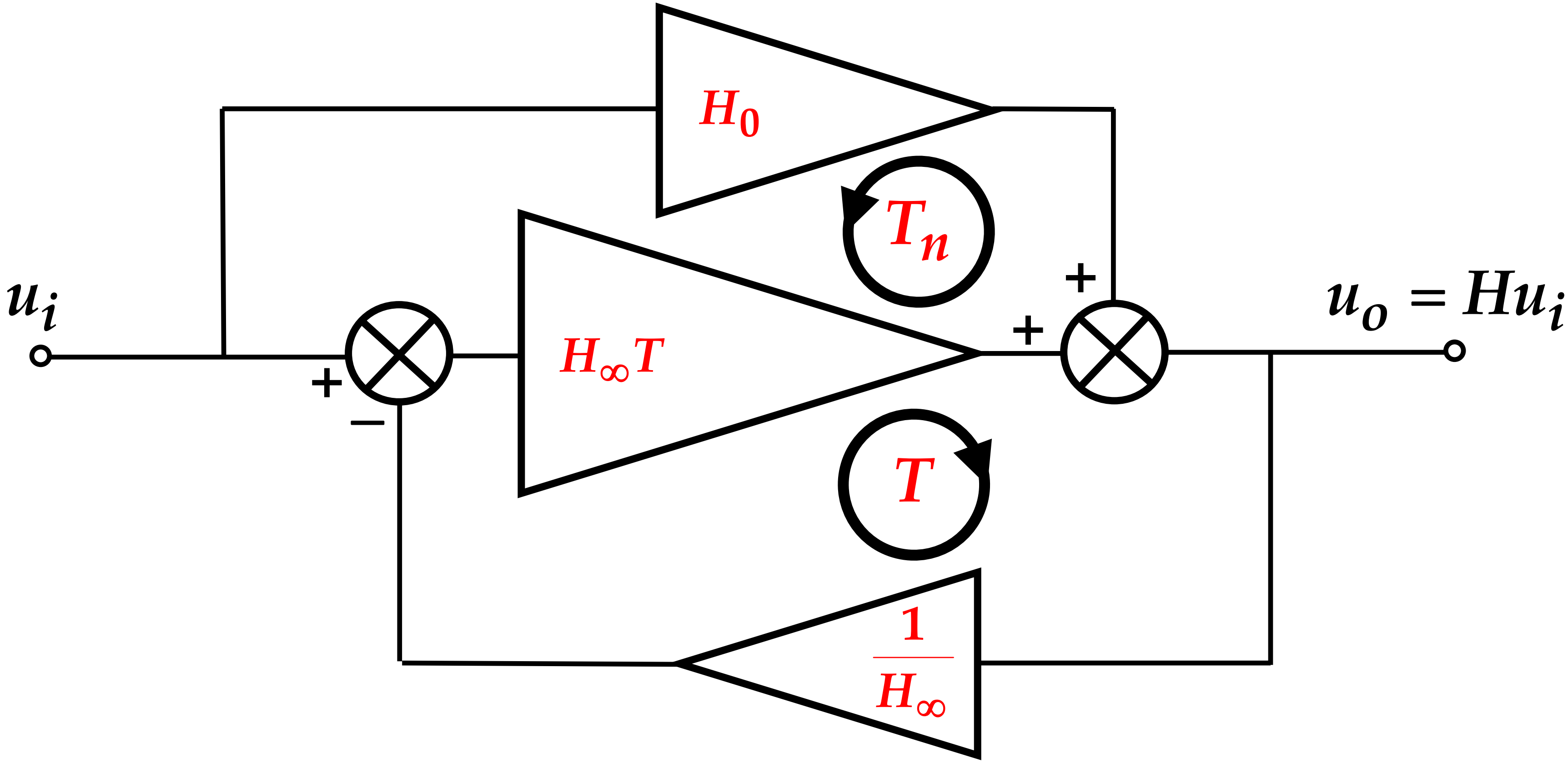
first level TF — second level TFs

$$H \equiv \left.\frac{u_o}{u_i}\right|_{u_z=0} \qquad H_\infty \equiv \left.\frac{u_o}{u_i}\right|_{u_y=0} \qquad H_0 \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \qquad T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \qquad T_n \equiv \left.\frac{u_y}{u_x}\right|_{u_o=0}$$

closed-loop gain — ideal closed-loop gain — direct forward transmission — loop gain — null loop gain

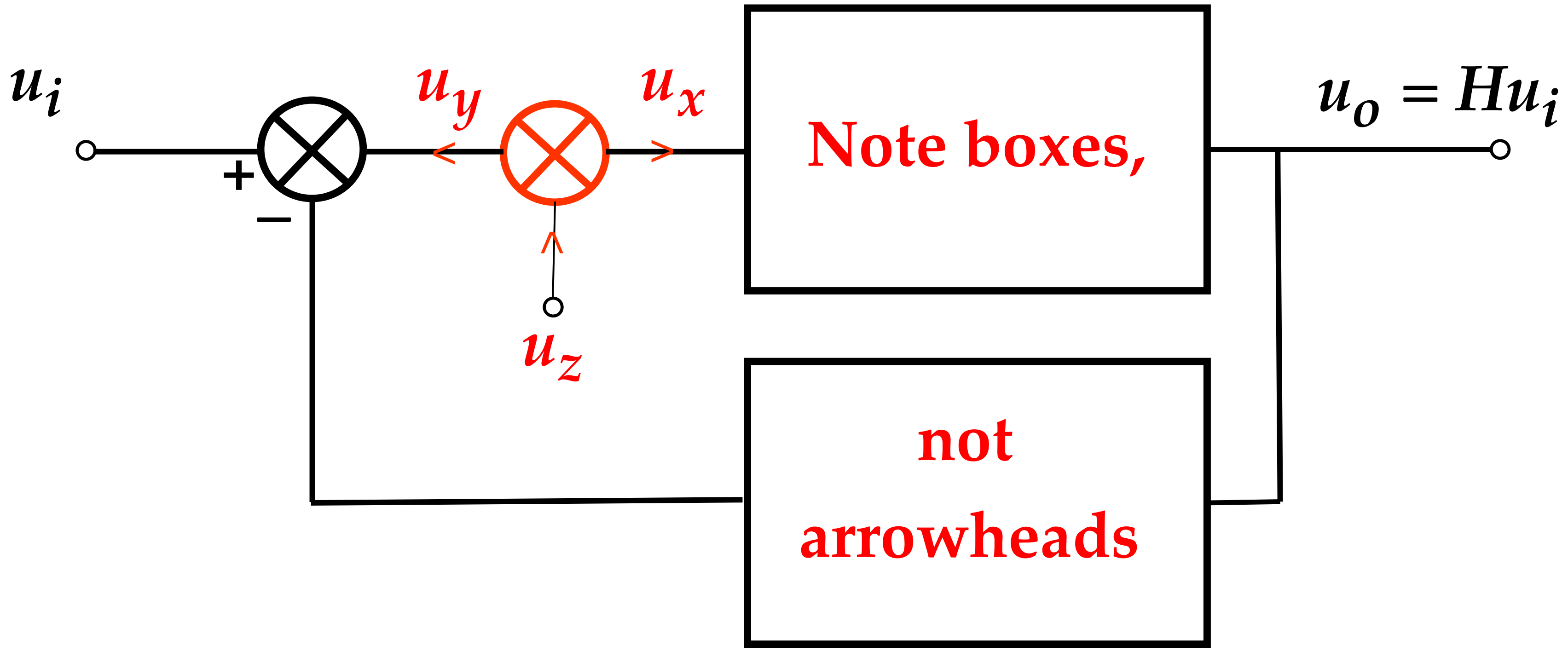
The augmented feedback block diagram:

This is the same block diagram that represents the conventional model, plus the H_0 block, and the corresponding null loop gain T_n, which represent *nonidealities* not accounted for in the conventional model.



One of these nonidealities is the direct forward transmission through the feedback path "in the wrong direction."

The GFT Approach:



Inject a test signal u_z at error summing point.

The GFT gives all the second-level TFs directly in terms of the circuit elements.

As already seen, the GFT can be expressed as the weighted sum of two components:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

It is useful to define the weighting factors as "discrepancy factors" D and D_0 :

$$D \equiv \frac{T}{1+T} \qquad\qquad D_0 \equiv \frac{1}{1+T}$$

Also define a "null discrepancy factor" $D_n \equiv 1+\frac{1}{T_n}$

$$H = H_\infty D D_n = H_\infty D + H_0 D_0$$

When $T >> 1, \quad D \to 1$ **and** $D_0 \to 1/T$ **When** $T_n >> 1, \quad D_n \to 1$

When $T << 1, \quad D \to T$ **and** $D_0 \to 1$ **When** $T_n << 1, \quad D_n \to 1/T_n$

Different versions of the GFT:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

$$H = H_\infty D D_n = H_\infty D + H_0 D_0$$

Thus, the direct forward transmisson nonideality H_0 appears either:

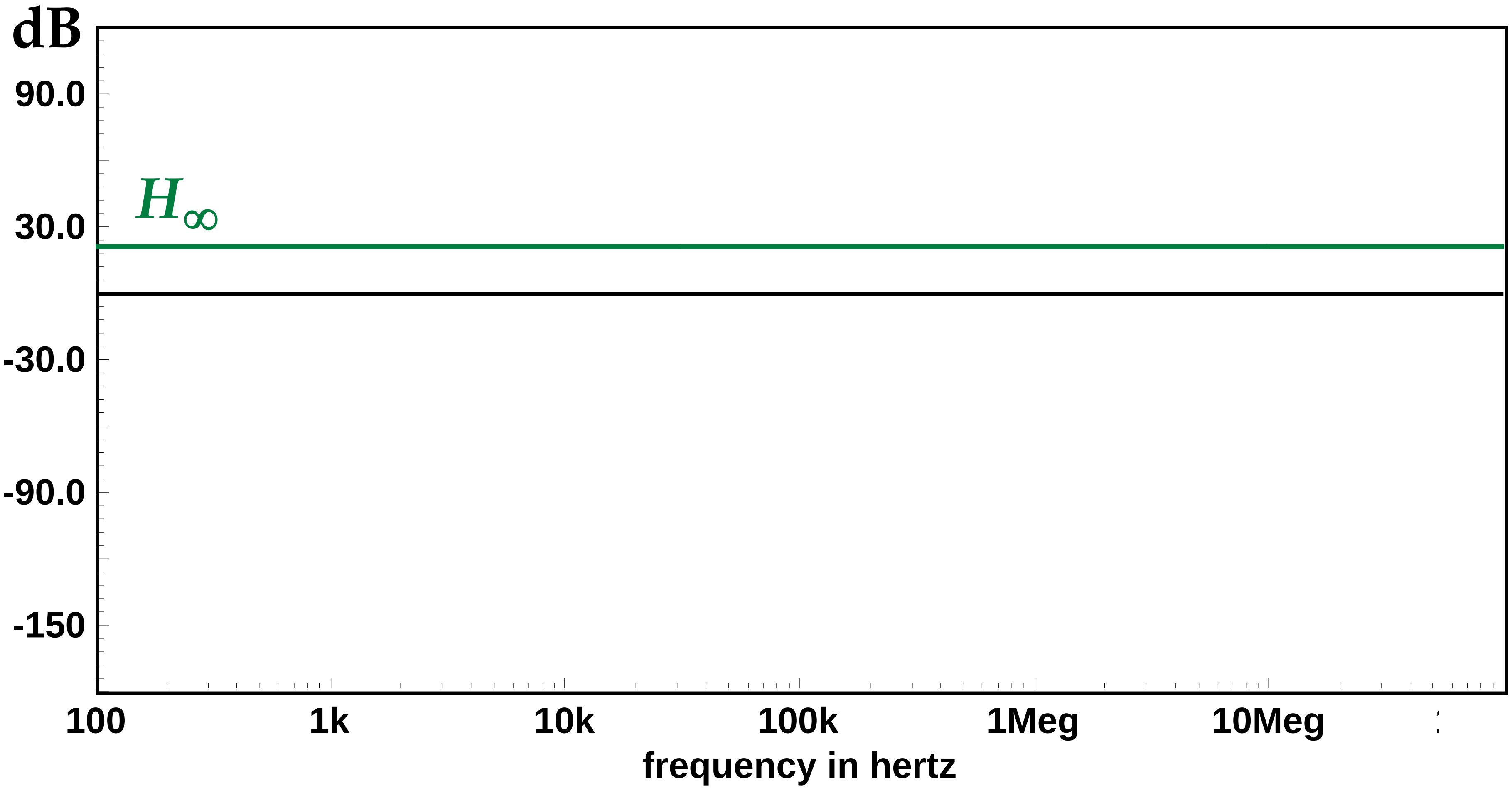
as a multiplier term, indirectly via T_n or D_n, or

as an additive term, directly

The job of a *designer*, as distinct from that of an *analyst*, is to construct hardware that meets specifications within certain tolerances.

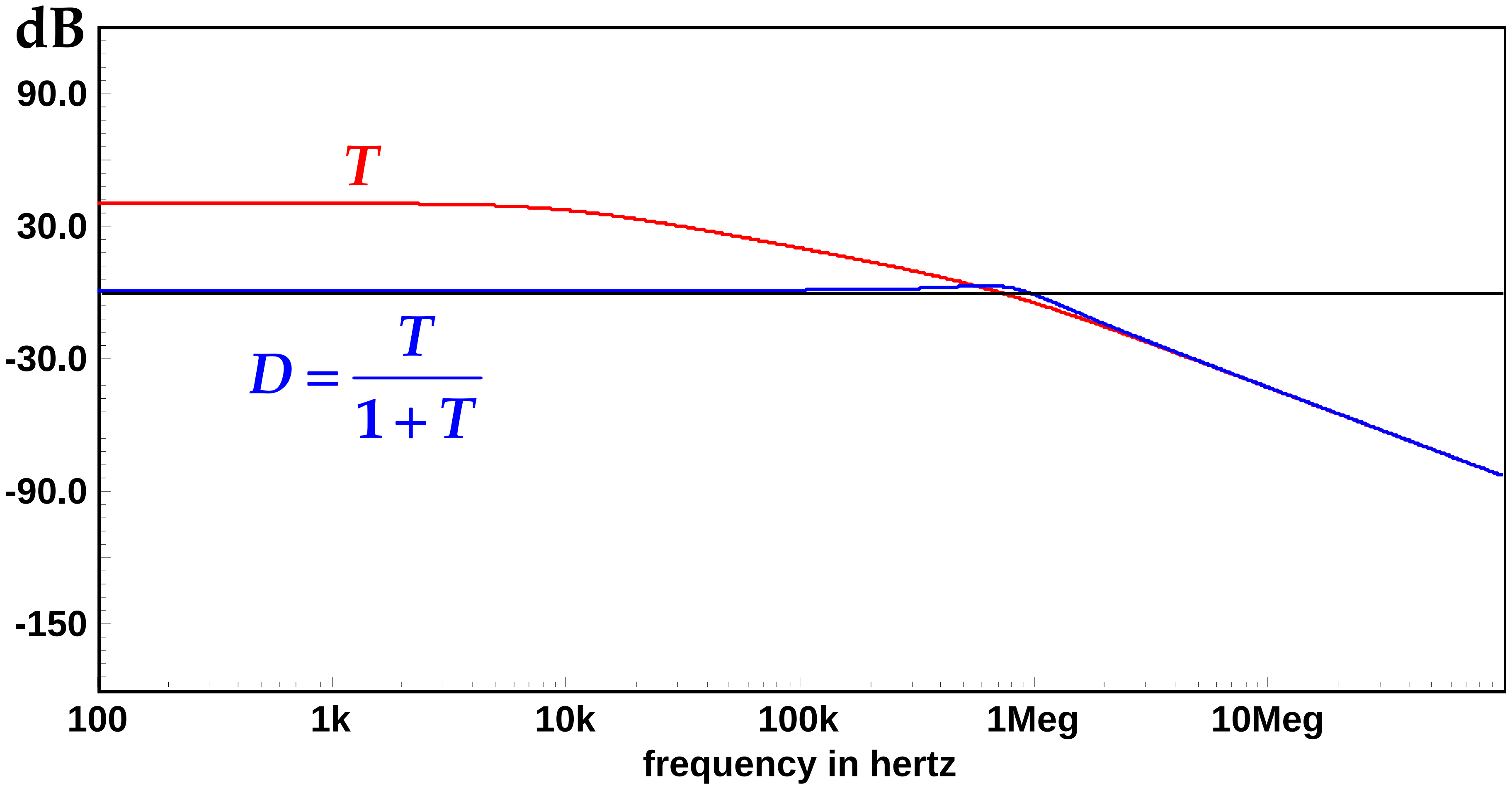
If you are designing a feedback amplifier, you effectively proceed through four steps:

Design Step #1



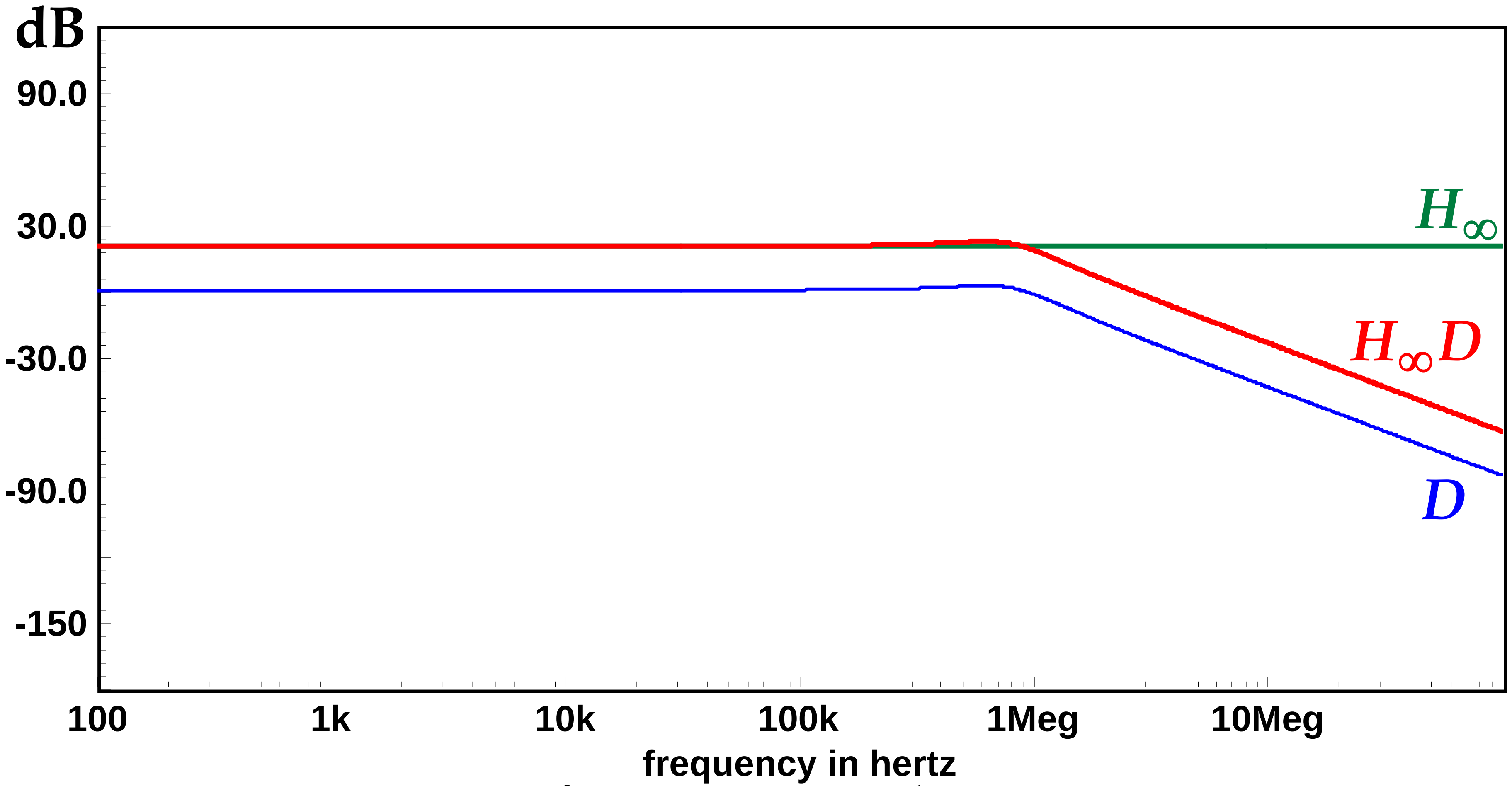
Design the feedback network $K = 1/H_\infty$ so that H_∞ meets the specification

Design Step #2



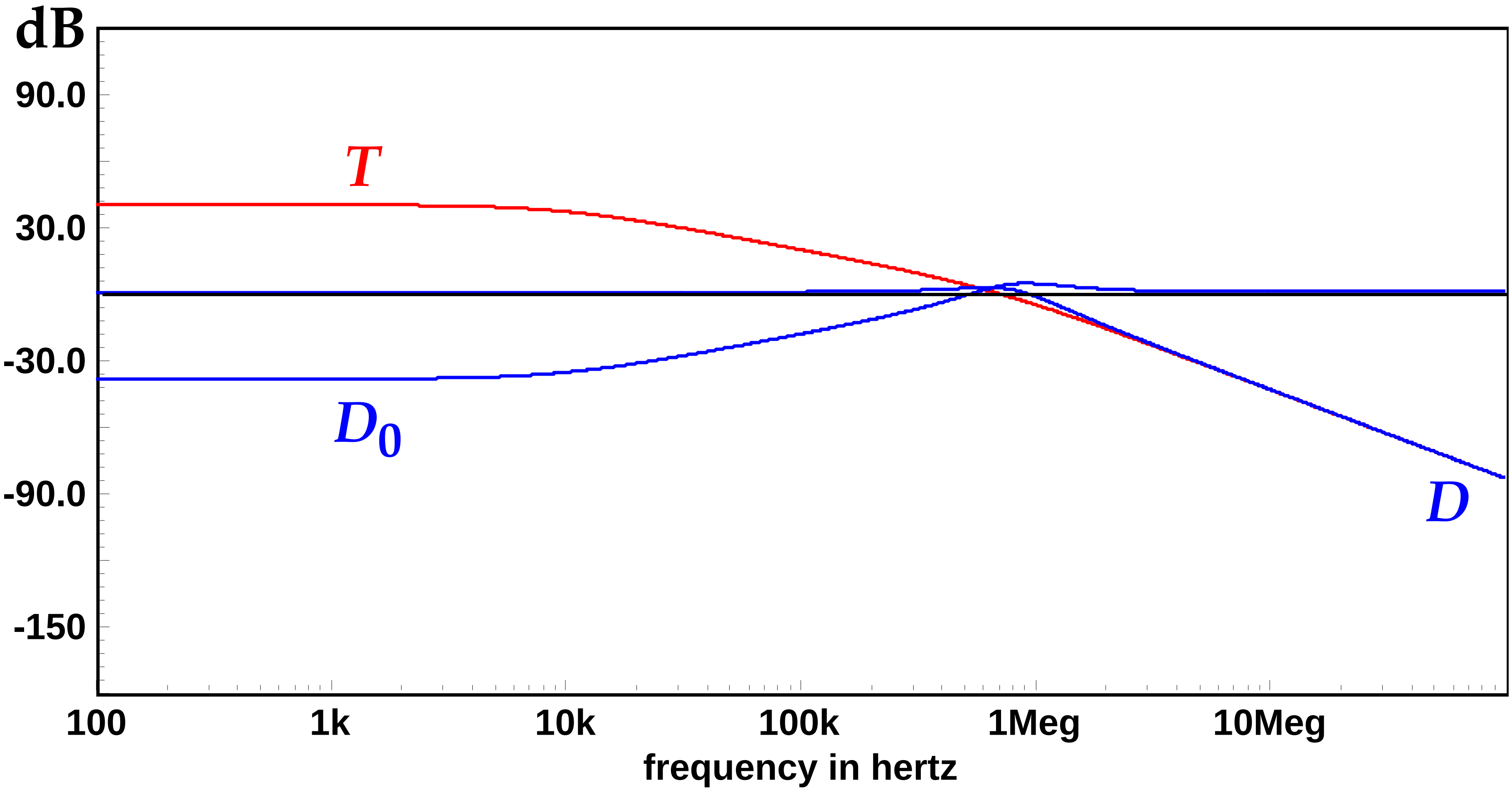
Design the loop gain so that T is large enough that the actual H meets the specification within the allowed tolerances

Design Step #2 cont.



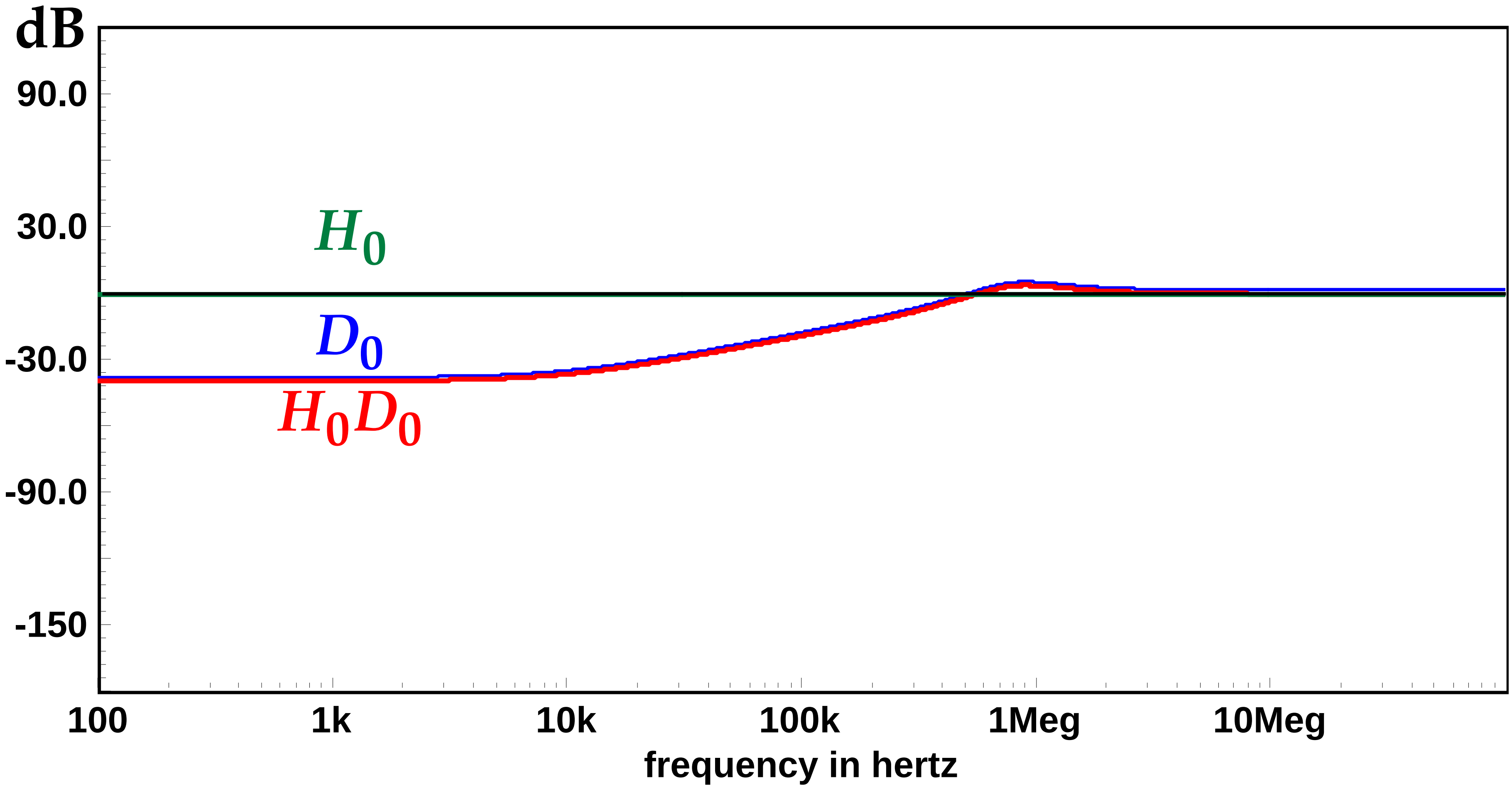
Construct $H_\infty D$ from H_∞ and D

Design Step #3



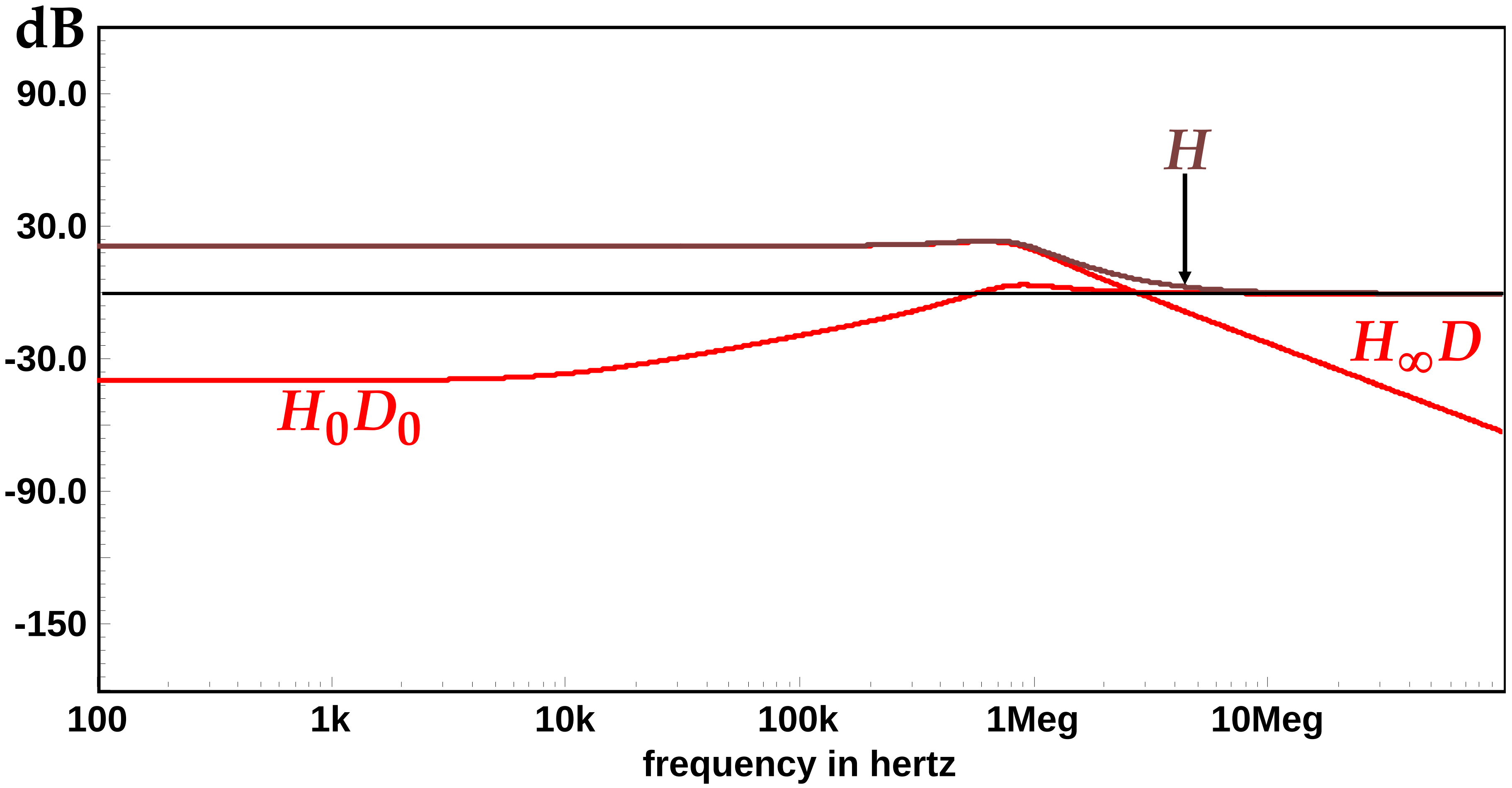
Design *T* to be large enough up to a sufficiently high frequency. Usually the most difficult step; includes the stability requirement

Design Step #4

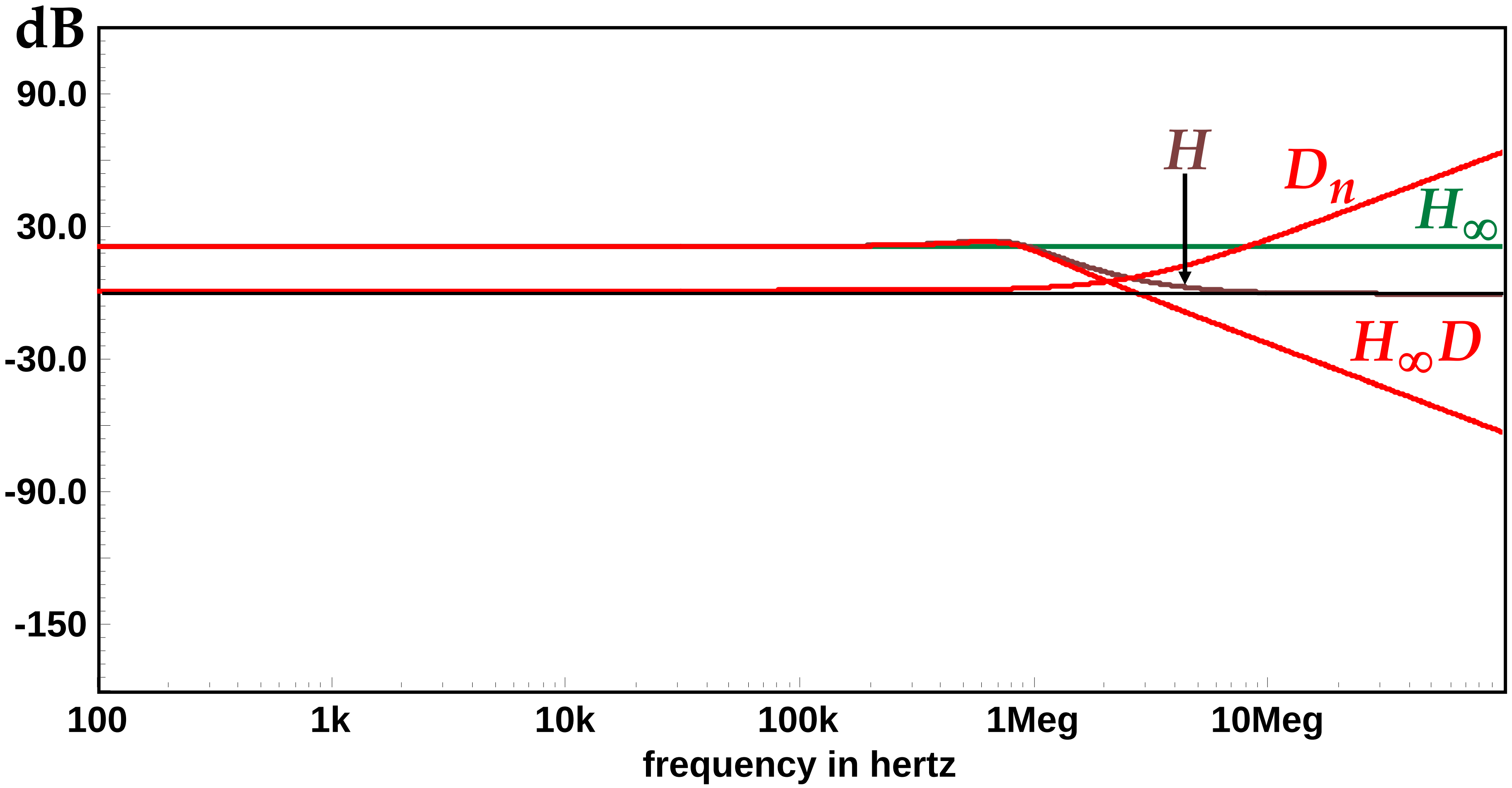


Design the direct forward transmission so that H_0 is small enough that the actual H meets the specification within the allowed tolerances

Design Step #4 cont.



Construction of H with H_0D_0 as an additive term to $H_\infty D$



Construction of H with D_n as a multiplier term to $H_\infty D$

Calculation of the second level TFs H_∞, T, H_0, T_n by injection of a test signal u_z.

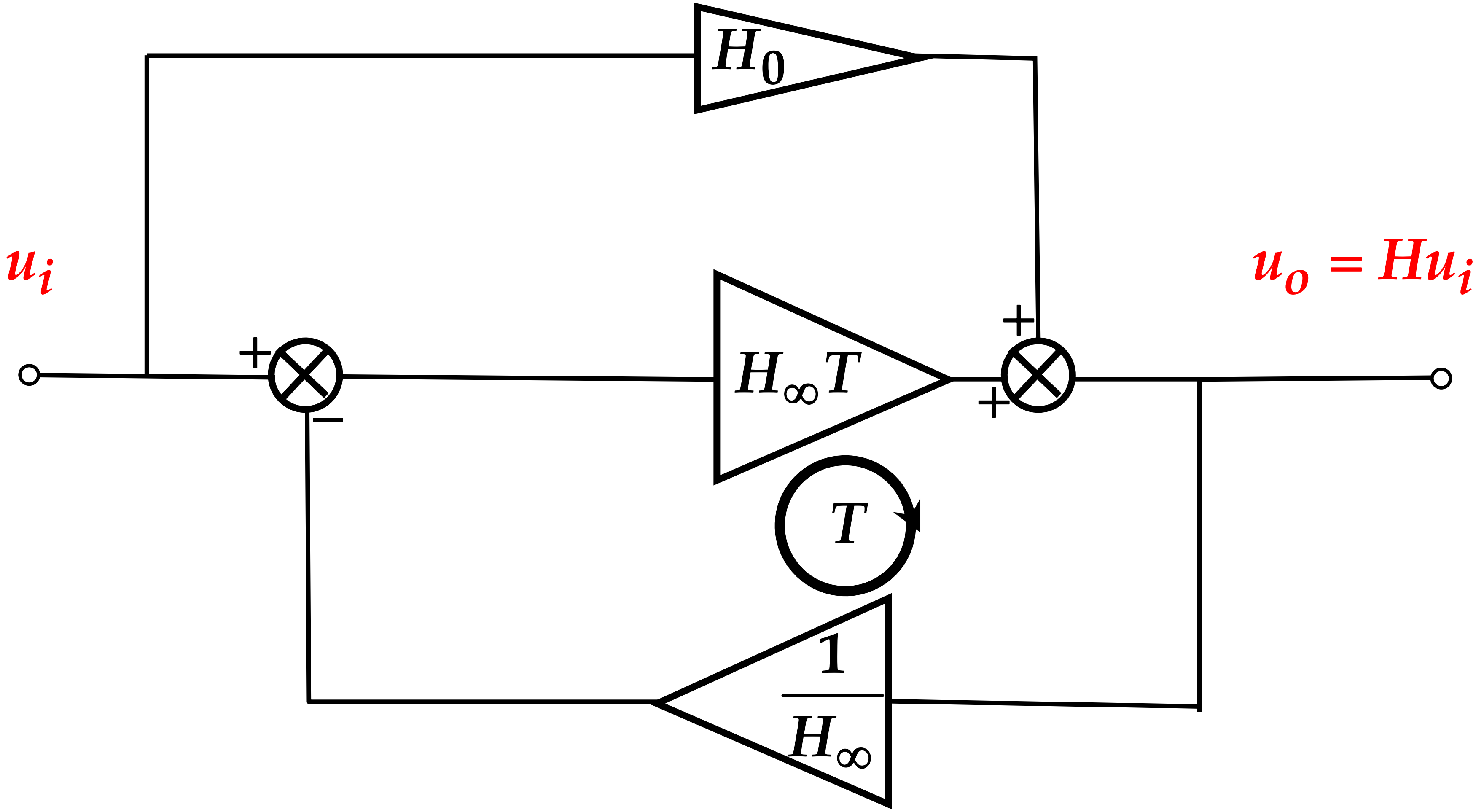
Ultimately, we want to calculate the second-level TFs directly from the circuit model, but first we'll do it from the block diagram.

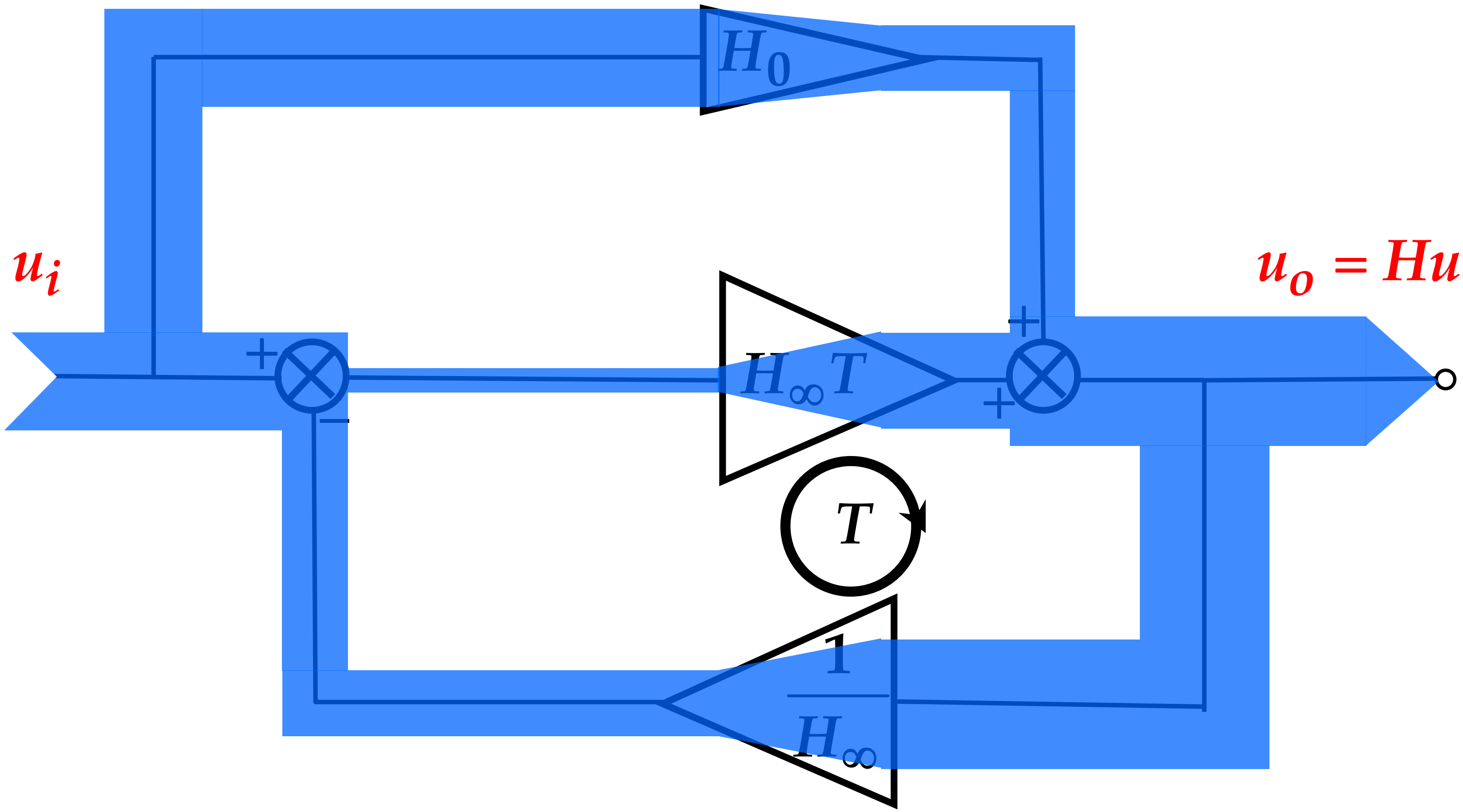
We want to do this by *signal injection*, without disturbing the circuit configuration and therefore without disturbing the circuit determinant.

Normally, we inject a single signal and calculate a TF whose input is that signal. This is single injection (si).

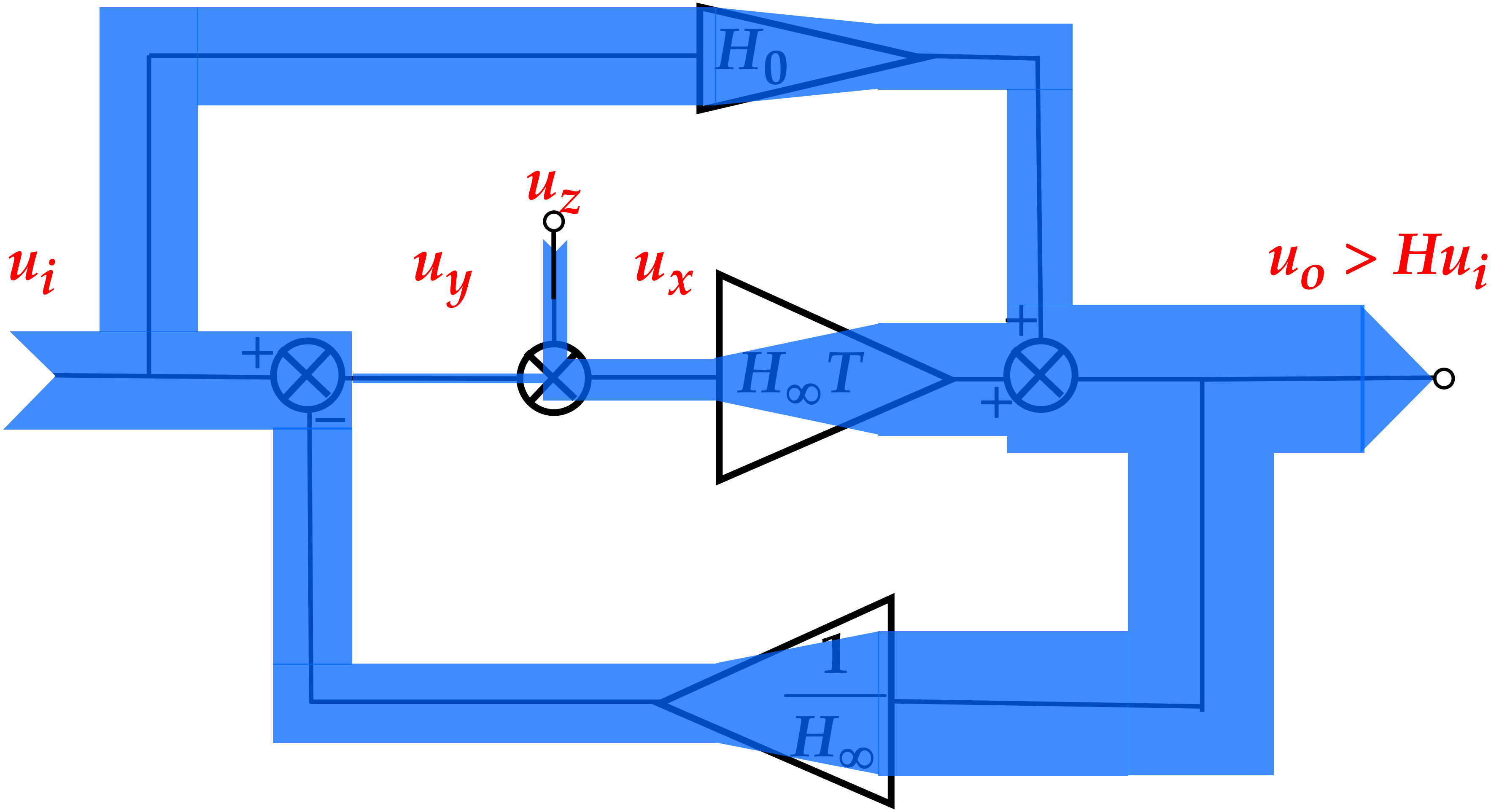
However, a powerful analytic technique is to inject two signals, mutually adjust them to null some dependent signal, and calculate a TF whose input is one or the other of the two injected signals. This is *null double injection* (ndi).

Consider the second-level TF H_∞, the ideal closed-loop gain. The actual closed-loop gain H falls short of H_∞ because the error signal, which is the difference between the input signal u_i and the fedback signal Ku_o, is not zero when the loop gain is not infinite.

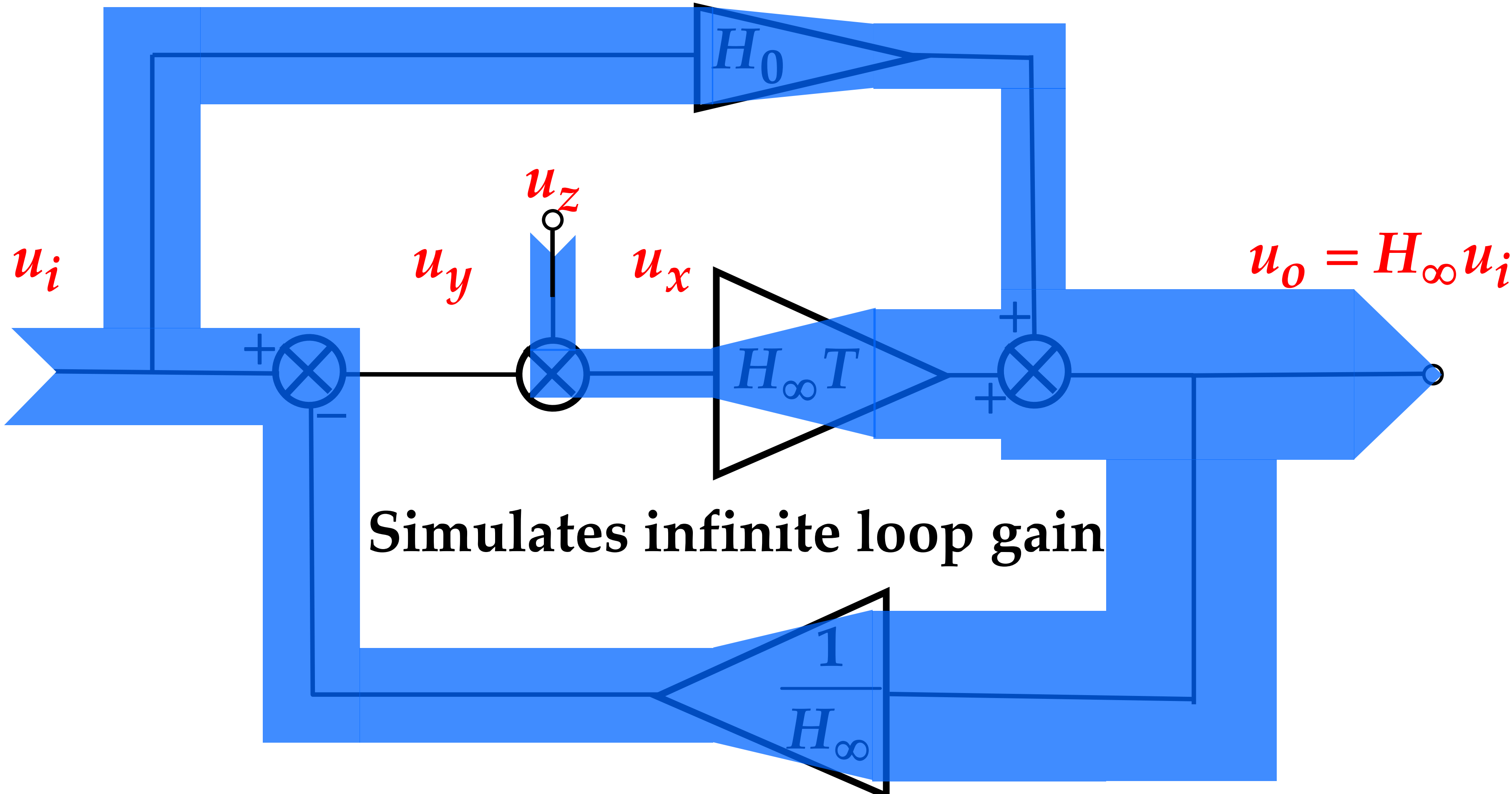

H_0
u_i
$u_o = Hu_i$
+
+
+
−
$H_\infty T$
T
$\frac{1}{H_\infty}$



Normal closed-loop operation

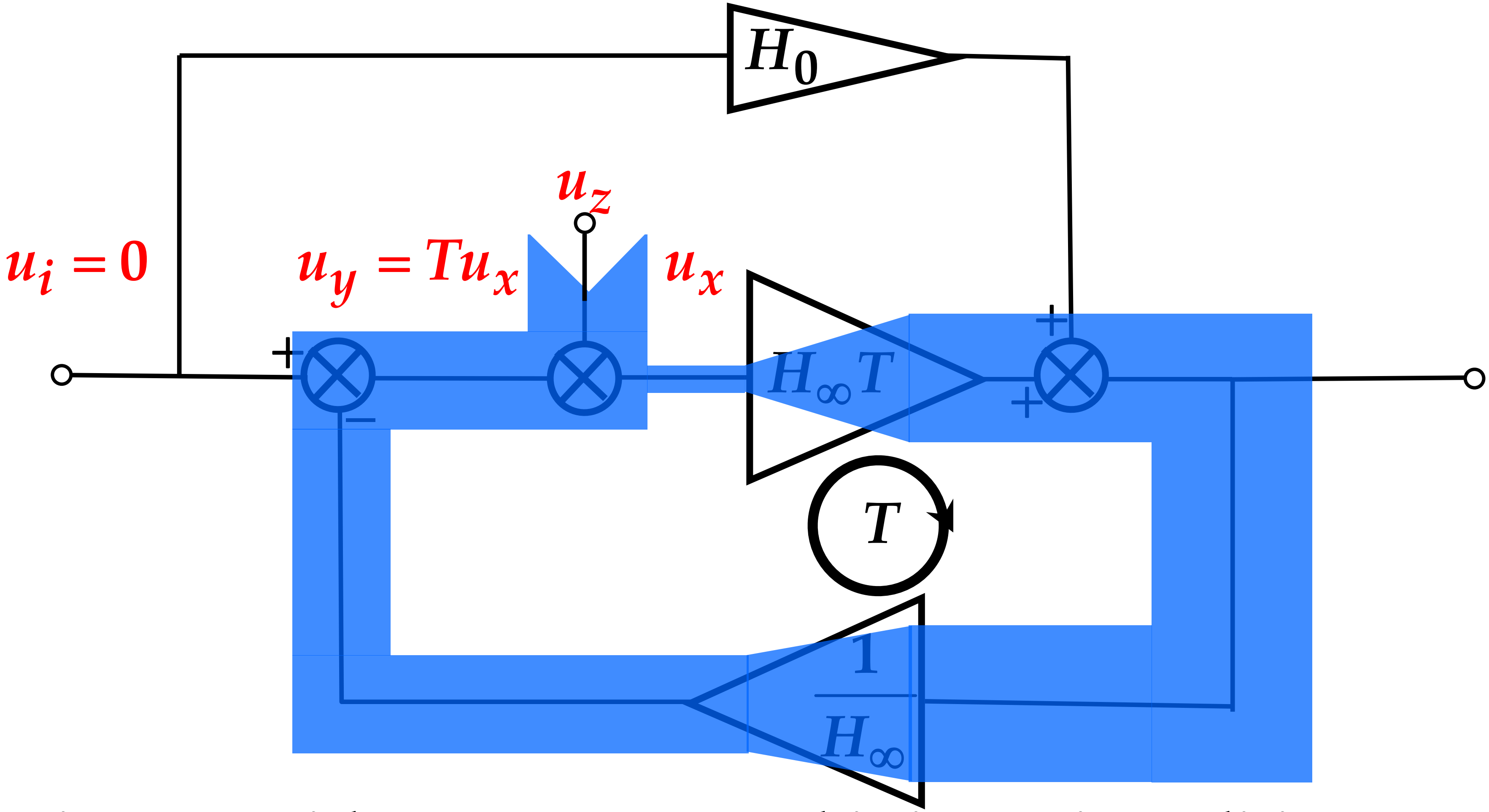


Inject a test signal u_z that adds to the error signal: the output u_o increases and the error signal u_y decreases



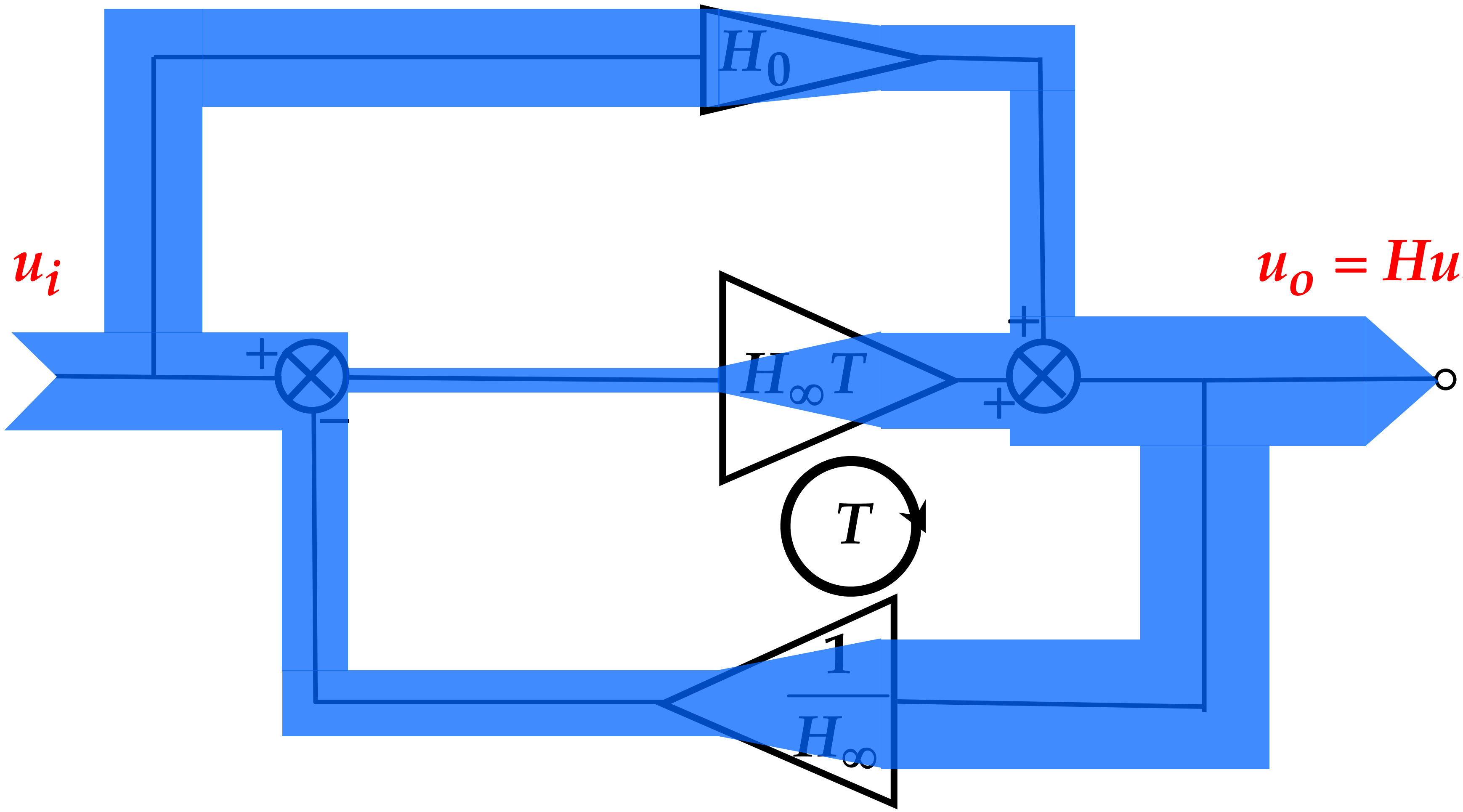
Increase u_z until u_y falls to zero: this is an ndi *condition*.

The ndi *calculation* is $H_\infty = \left.\frac{u_o}{u_i}\right|_{u_y=0}$

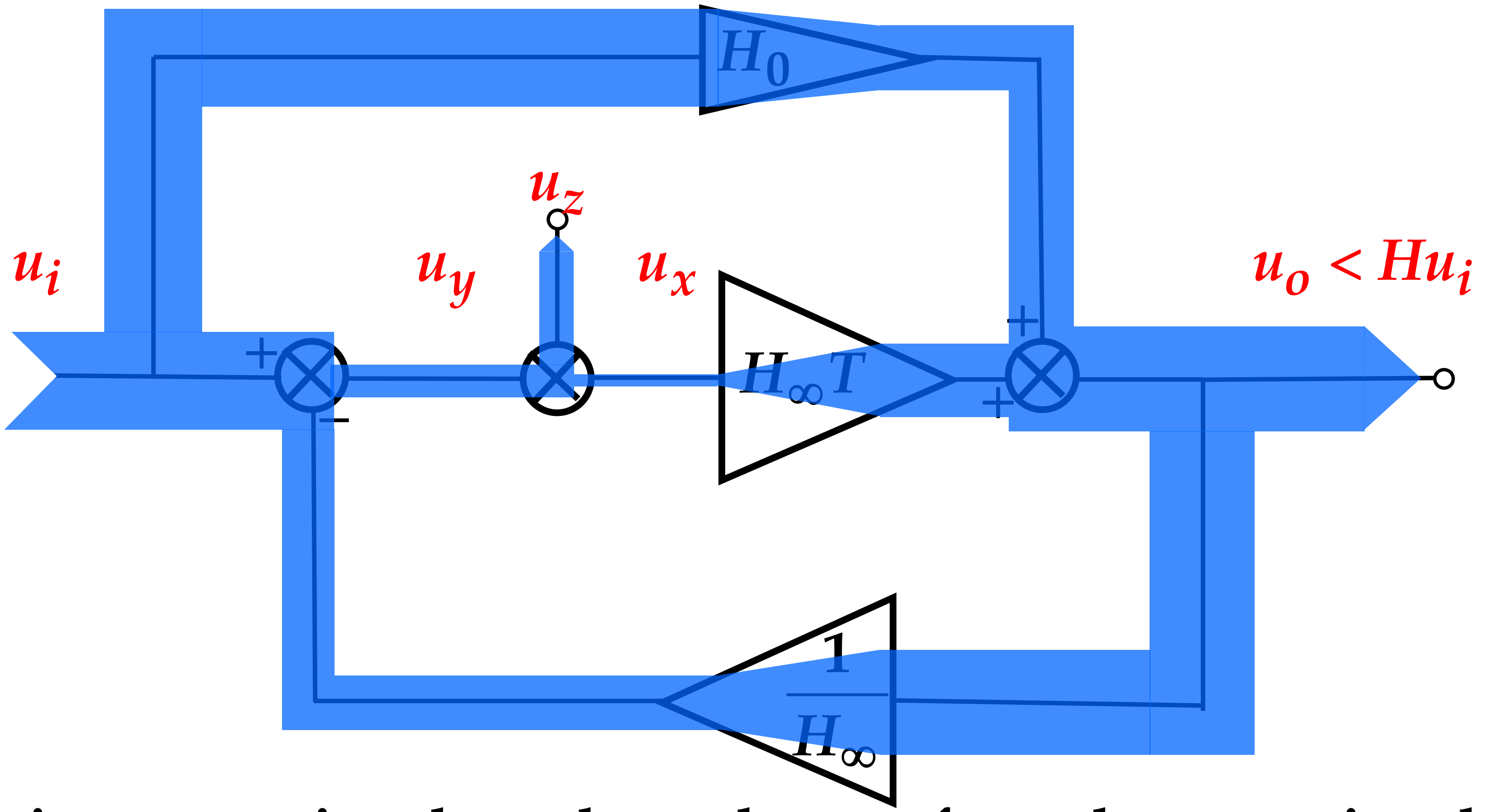


Inject u_z with u_i set to zero: this is an si condition.

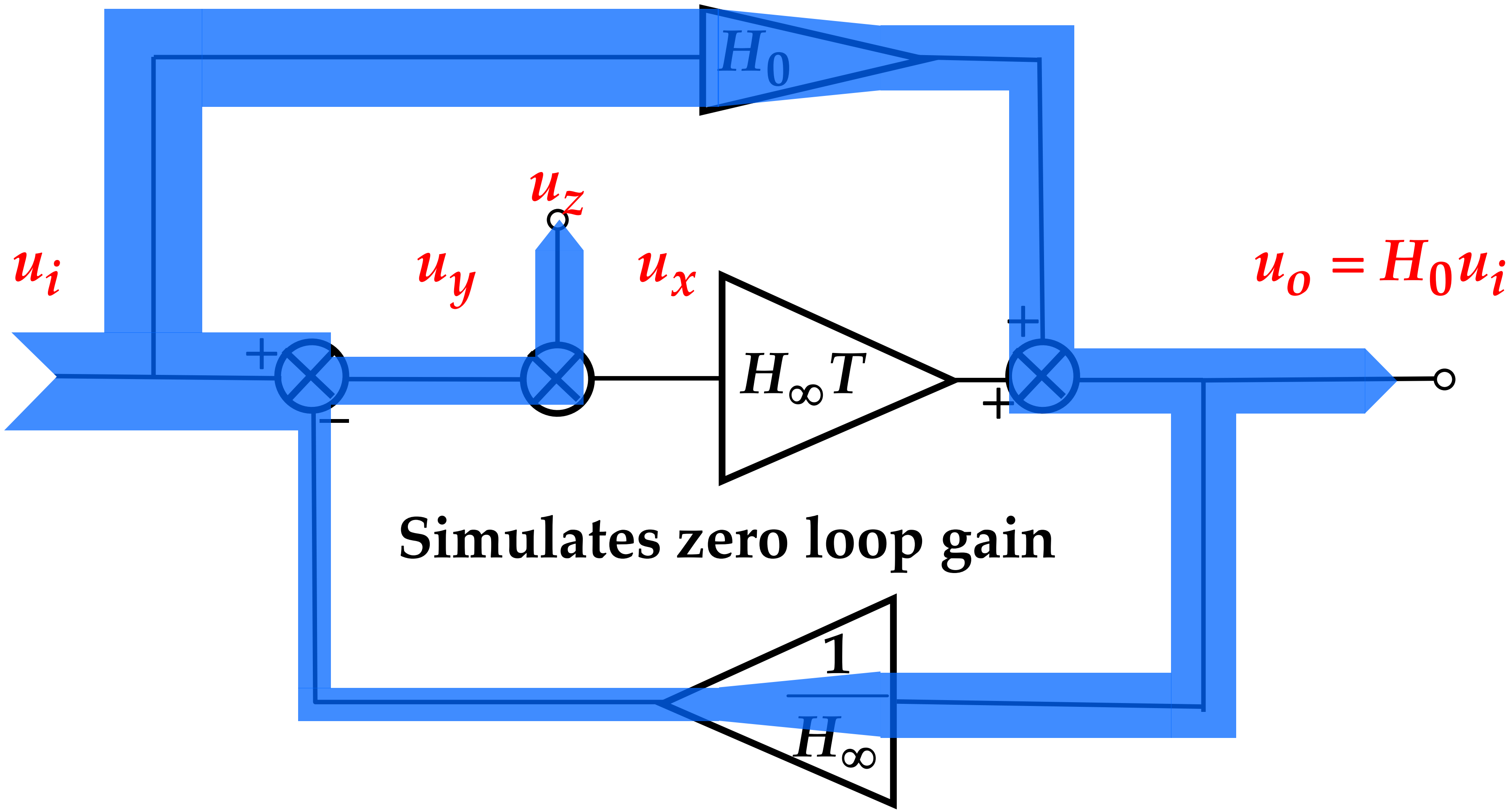
The si *calculation* is $T = \left.\dfrac{u_y}{u_x}\right|_{u_i=0}$



Back to normal closed-loop operation

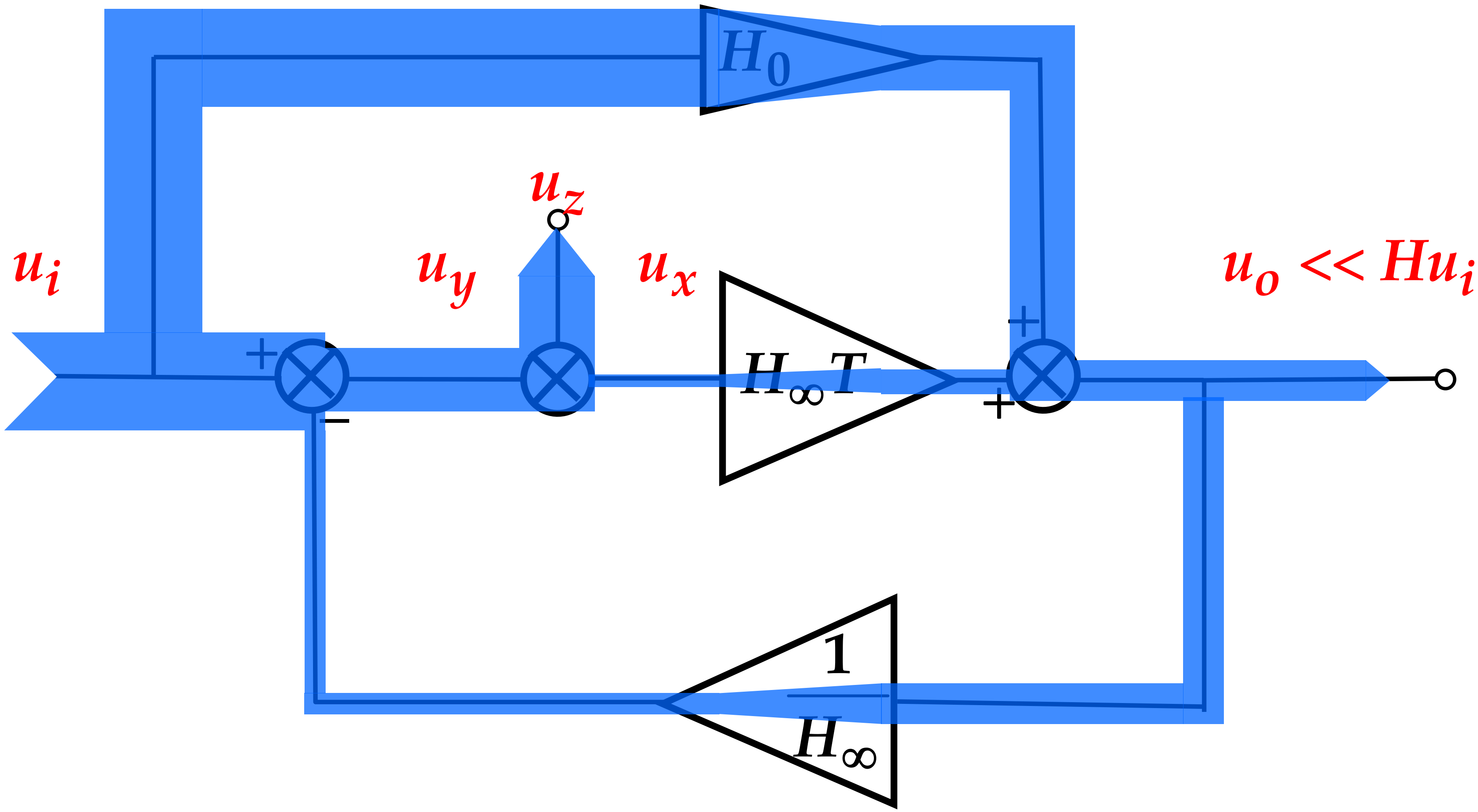


Inject a test signal u_z that subtracts from the error signal: the output u_o decreases and u_x decreases

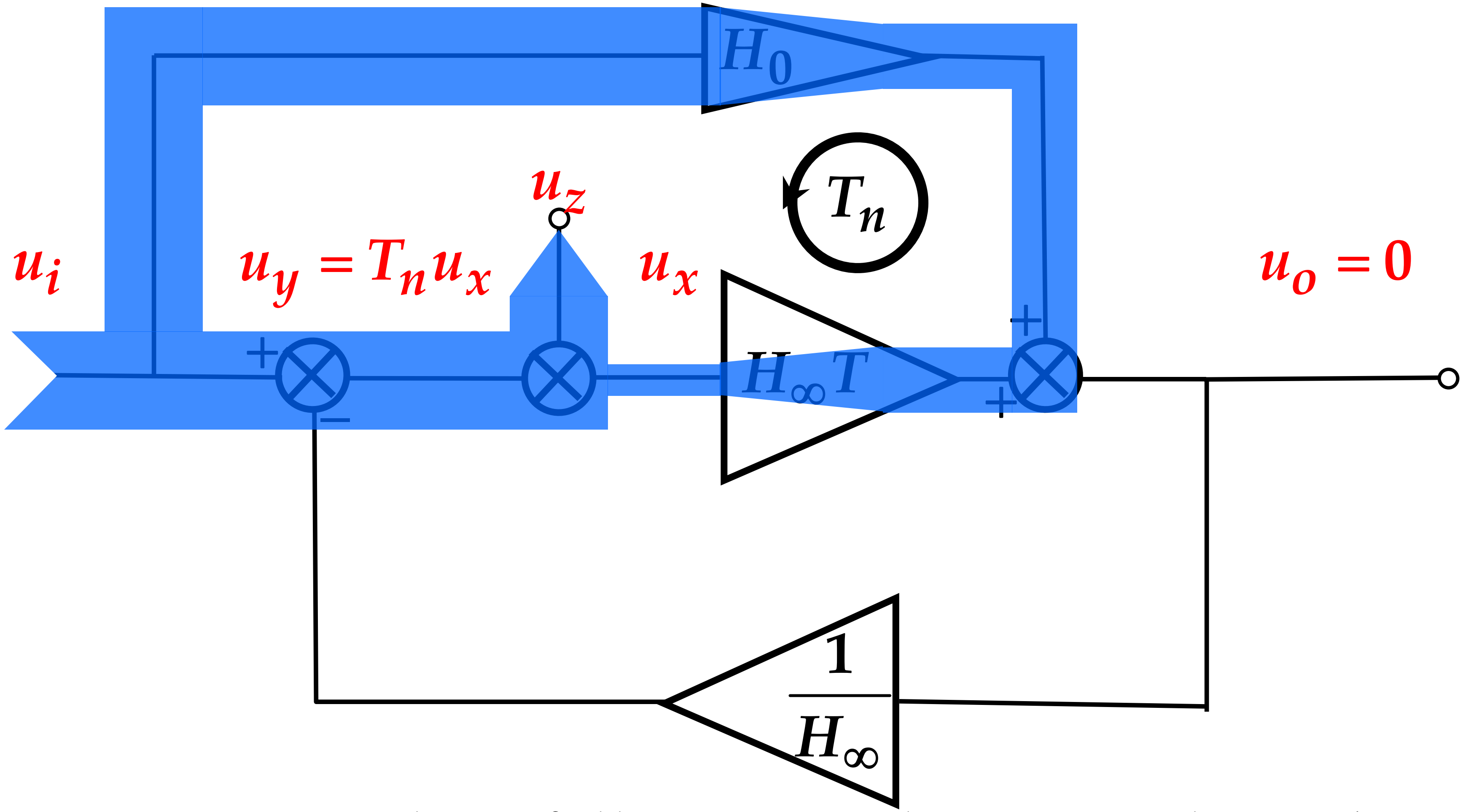


Increase u_z until u_x falls to zero: this is an ndi *condition*.

The ndi *calculation* is $H_0 = \left.\dfrac{u_o}{u_i}\right|_{u_x=0}$

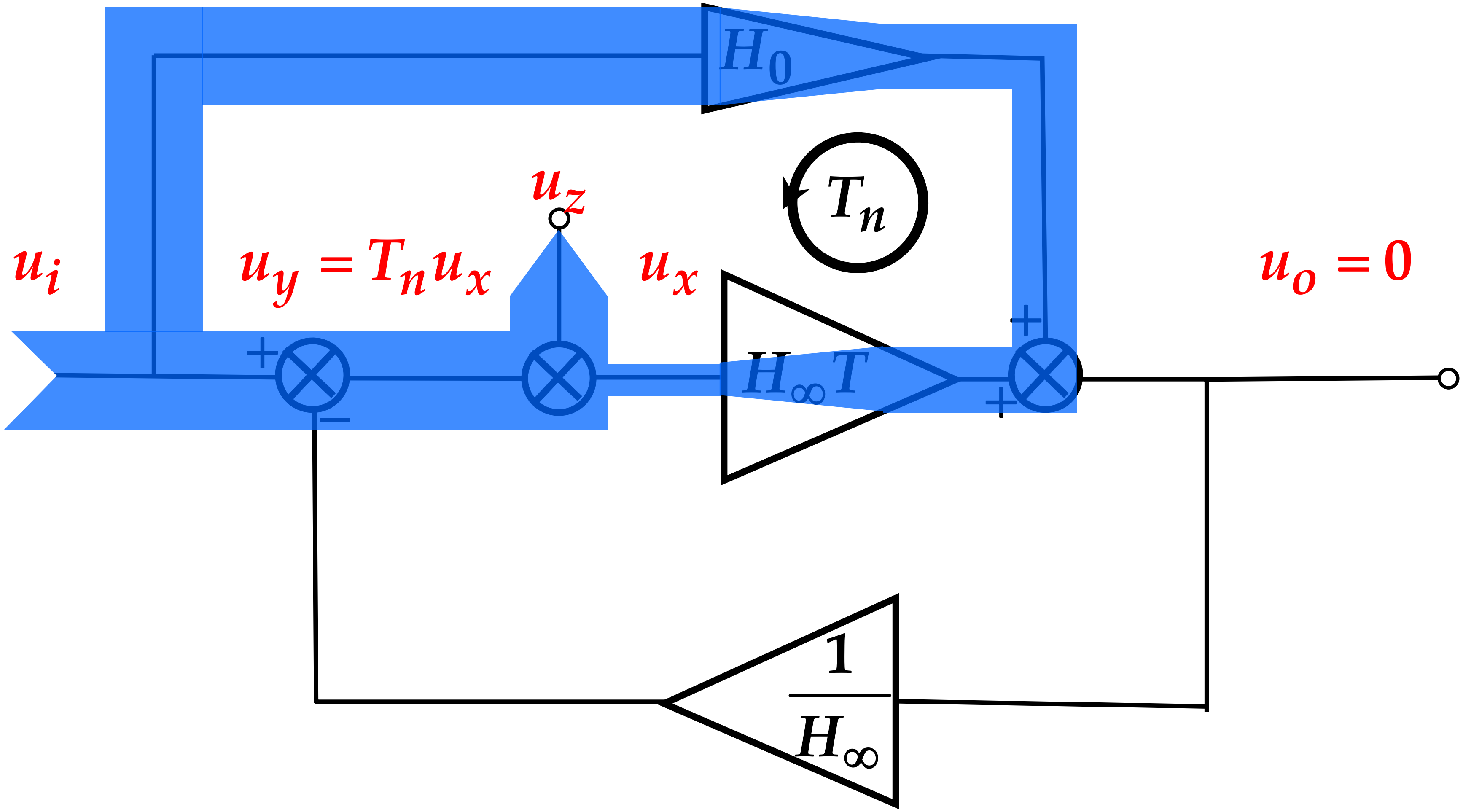


Further increase of u_z causes u_x to reverse, and a further drop in u_o



Increase u_z until u_o falls to zero: this is an ndi *condition*.

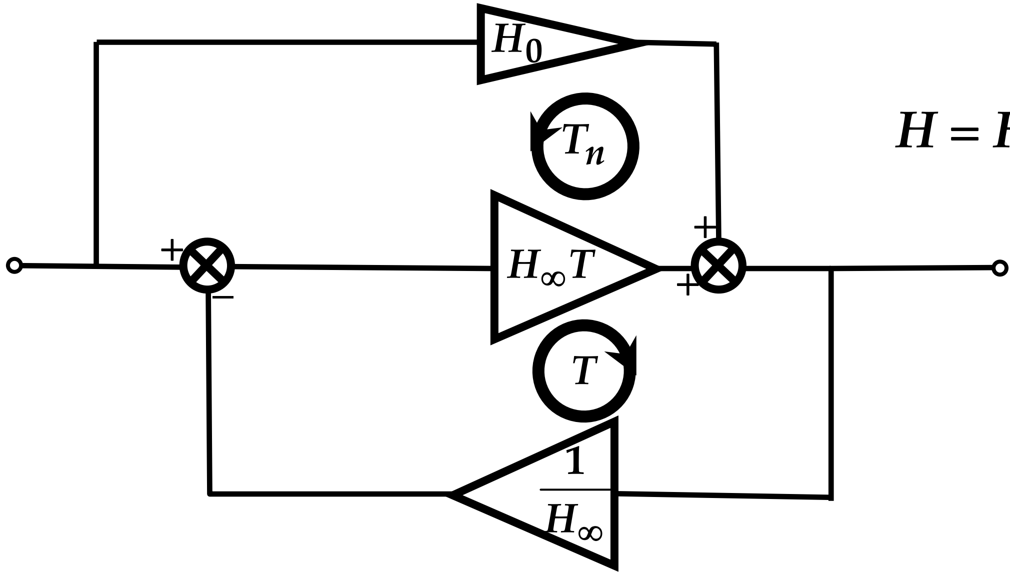
The ndi *calculation* is $T_n = \left.\frac{u_y}{u_x}\right|_{u_o=0}$



Under this ndi condition of nulled u_o,

$H_\infty T u_x = H_0 u_i$. **But** $u_y = u_i$, **so** $u_y / u_x = H_\infty T / H_0$,

which is how T_n **was originally defined.**



$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty \frac{1+\frac{H_0}{H_\infty T}}{1+\frac{1}{T}}$$

Redundancy relation:

$$\frac{T_n}{T} = \frac{H_\infty}{H_0}$$

ideal closed-loop gain: $H_\infty = \left.\frac{u_o}{u_i}\right|_{u_y=0}$ **ndi calculation**

principal loop gain: $T = \left.\frac{u_y}{u_i}\right|_{u_i=0}$ **si calculation**

direct fwd transmission: $H_0 = \left.\frac{u_o}{u_i}\right|_{u_x=0}$ **ndi calculation**

null loop gain: $T_n = \left.\frac{u_y}{u_x}\right|_{u_o=0}$ **ndi calculation**

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_∞, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_∞, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

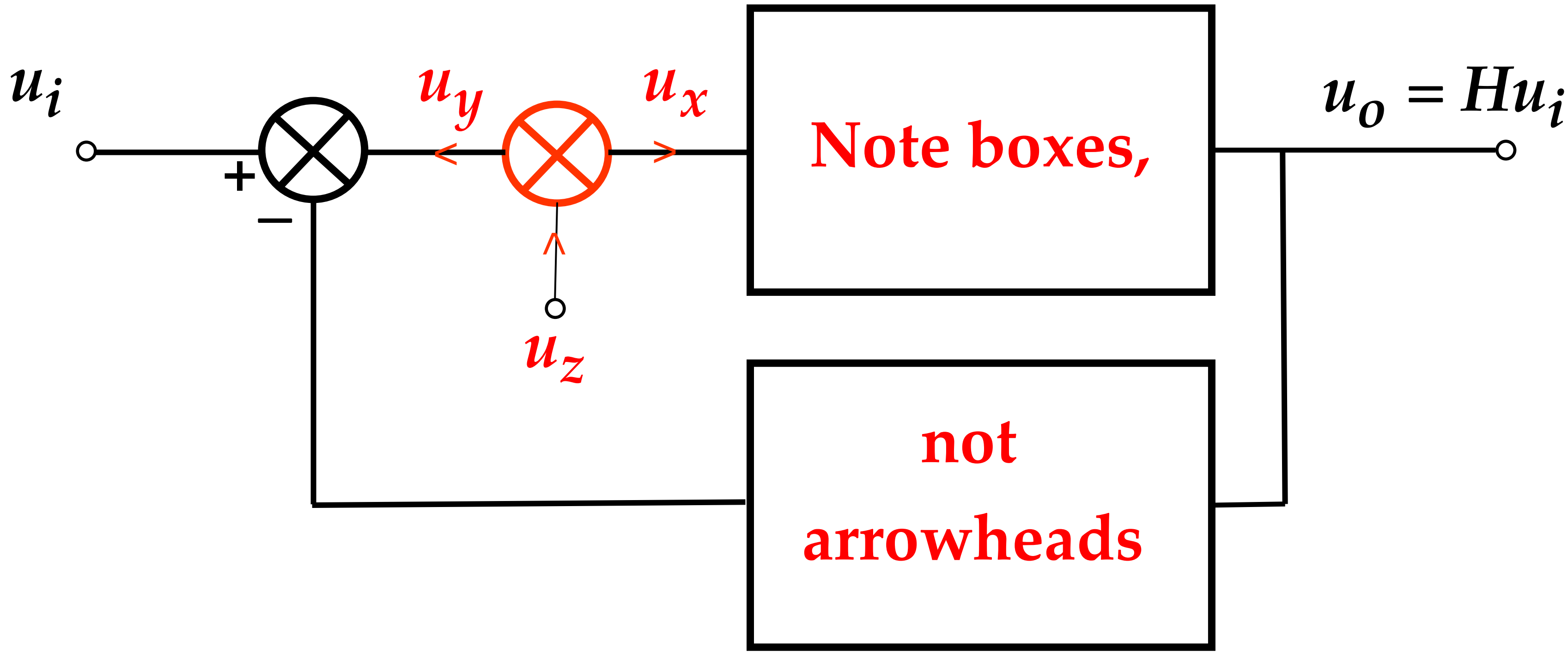
1. The test signal must be injected so that u_y is the error signal. This makes H_∞ equal to the Ideal Closed Loop Gain $1/K$, the reciprocal of the feedback ratio.

Test Signal Injection Configuration

In order for the *definitions* of the four second-level TFs H_∞, T, H_0, T_n to have the *interpretations* shown, it is necessary that the Test Signal Injection Configuration satisfy the conditions adopted in the preceding derivation:

2. The test signal must be injected inside the major loop, but outside any minor loops. This makes T represent the Principal Loop Gain.

The GFT Approach:



Inject a test signal u_z at error summing point.
The GFT gives all the second-level TFs directly in terms of the circuit elements.

Nonidealities:

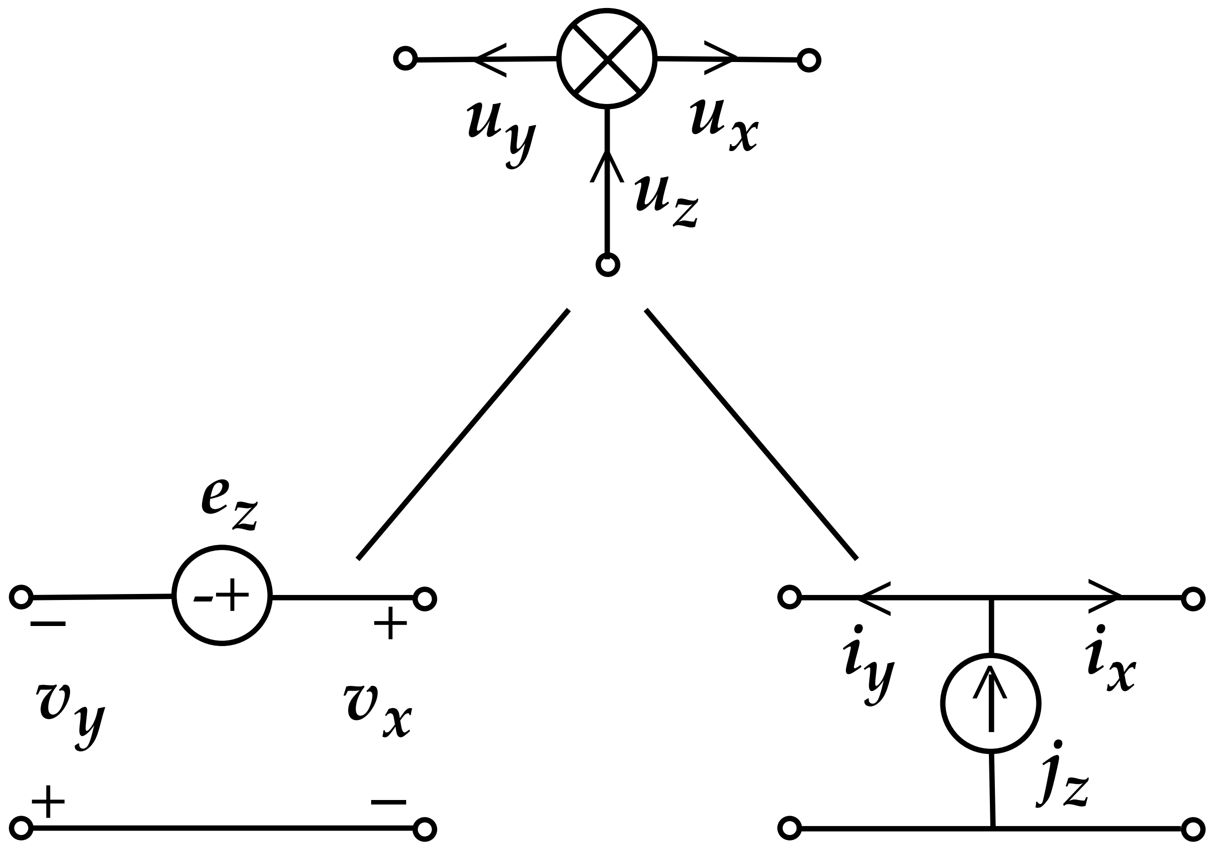
1. Reverse transmission through the feedback path

2. Reverse transmission through the forward path

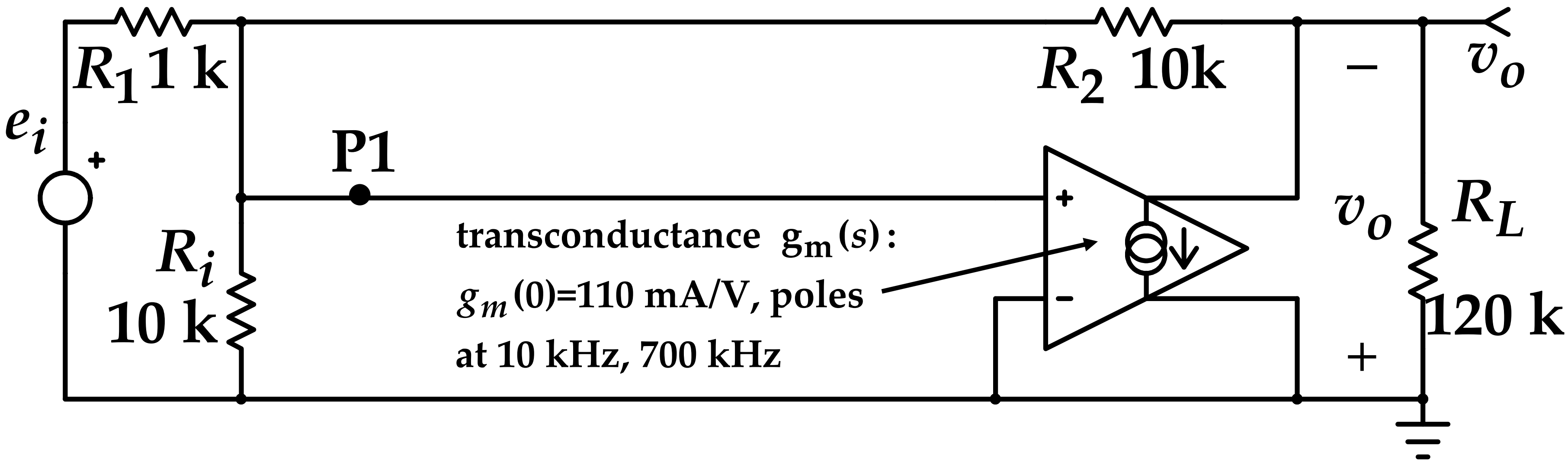
3. If both paths have reverse transmission, there is a nonzero reverse loop gain

All nonidealities are automatically accounted for by the GFT

Are u_x, u_y, u_z voltages or currents?

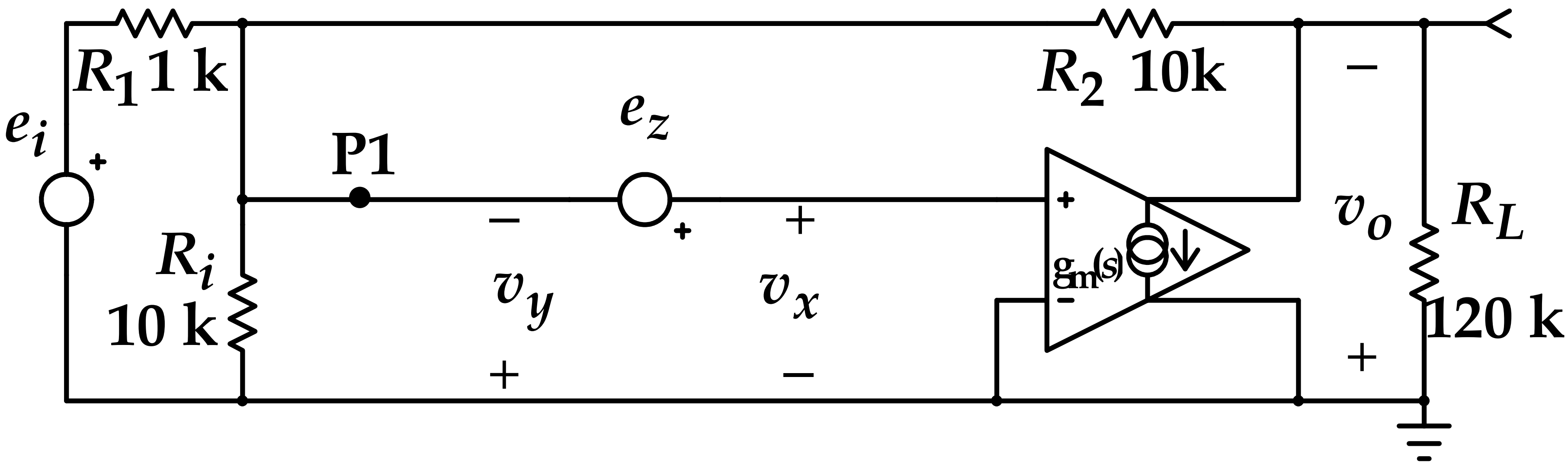


Inverting Opamp



We want to get $H_\infty = \dfrac{R_2}{R_1}$, **so inject** e_z **to add to the error voltage at P1:**

Ideal voltage injection point

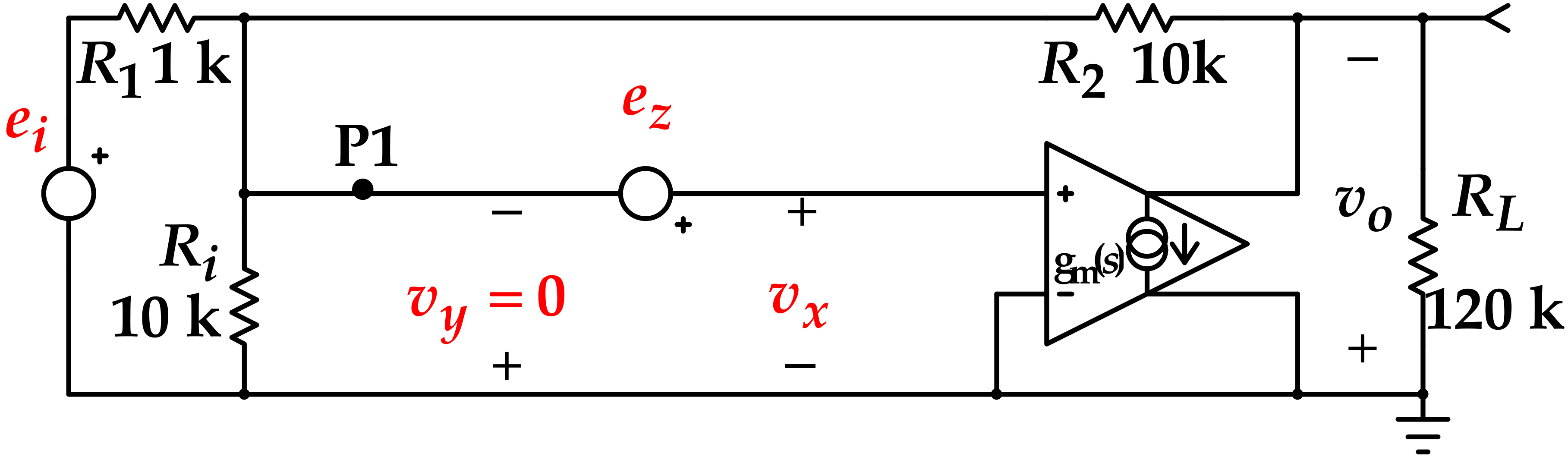


An ideal voltage injection point is where v_y comes from an ideal (zero impedance) voltage generator,
or **where v_x looks into an infinite impedance.**

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

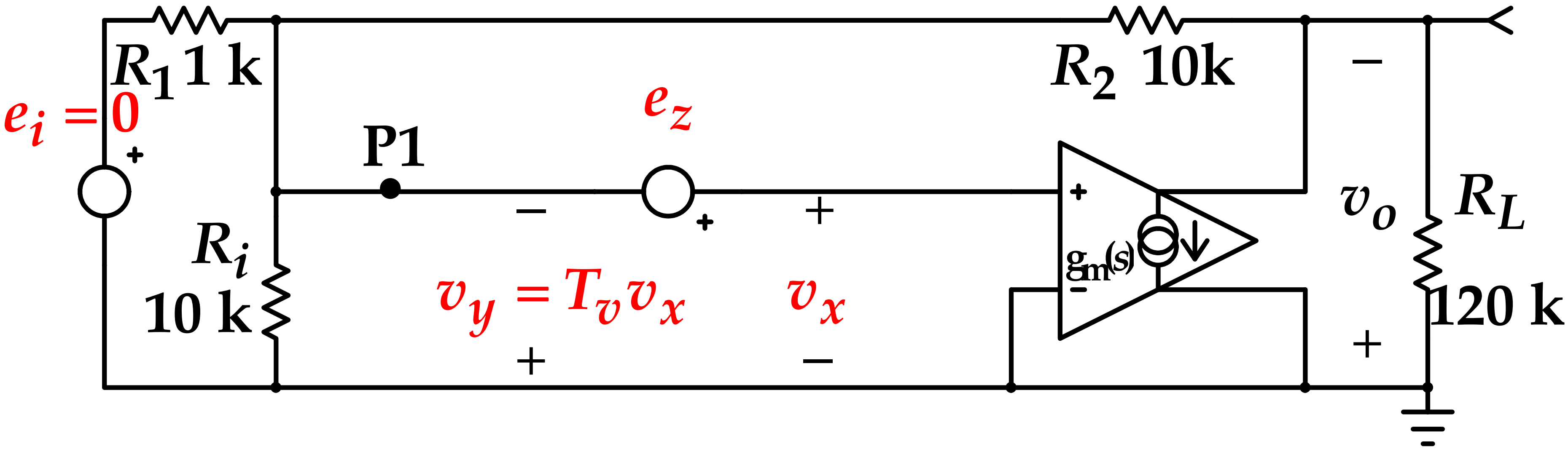
A superscript indicates the signal being nulled

$$v_o = H^{v_y} e_i$$



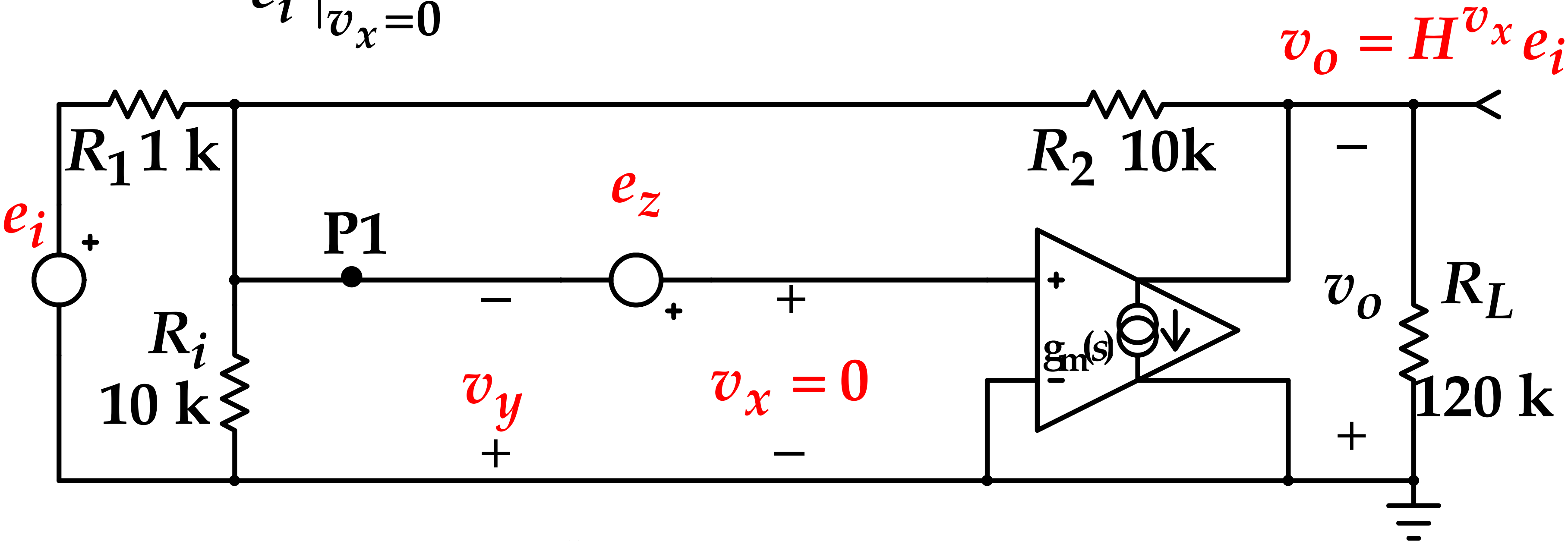
$$H^{v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20 \text{ dB}$$

$$T = \left.\frac{v_y}{v_x}\right|_{e_i=0} \equiv T_v$$



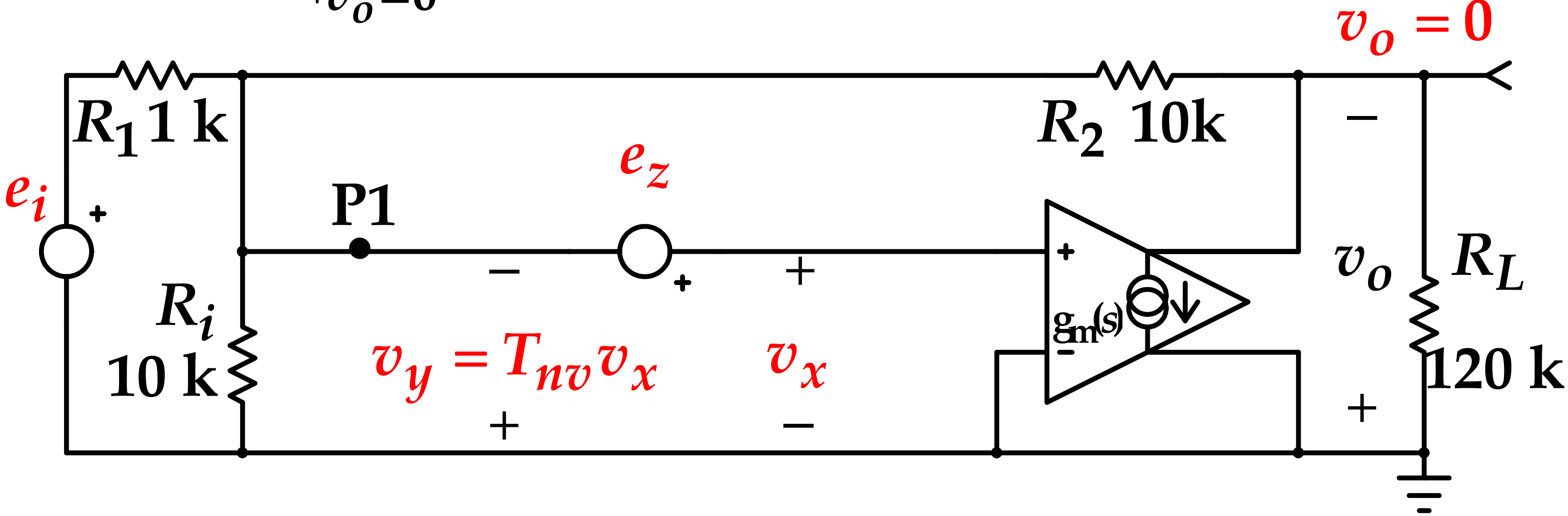
$$T_v(0) = g_m(0)[R_L \| (R_1 \| R_i + R_2)] \frac{R_1 \| R_i}{R_1 \| R_i + R_2} = 92 \Rightarrow 39.2 \text{ dB}$$

$$H_0 = \left.\frac{v_o}{e_i}\right|_{v_x=0} \equiv H^{v_x}$$



$$H^{v_x} = \frac{R_L}{R_2 + R_L} \frac{R_i \| (R_2 + R_L)}{R_1 + R_i \| (R_2 + R_L)} = 0.83 \Rightarrow -1.6 \text{ dB}$$

$$T_n = \left.\frac{v_y}{v_x}\right|_{v_o=0} \equiv T_{nv}$$



$$T_{nv}(0) = g_m(0) R_2 = 1100 \Rightarrow 60.8 \text{ dB}$$

The redundancy relation $\dfrac{T_{nv}(0)}{T_v(0)} = \dfrac{H^{v_y}}{H^{v_x}} = 12.0$

is verified.

Note that T_n is a simpler result from a shorter calculation than is H_0, which is usually the case.

Therefore, it may be preferable to use the second of the two versions of H:

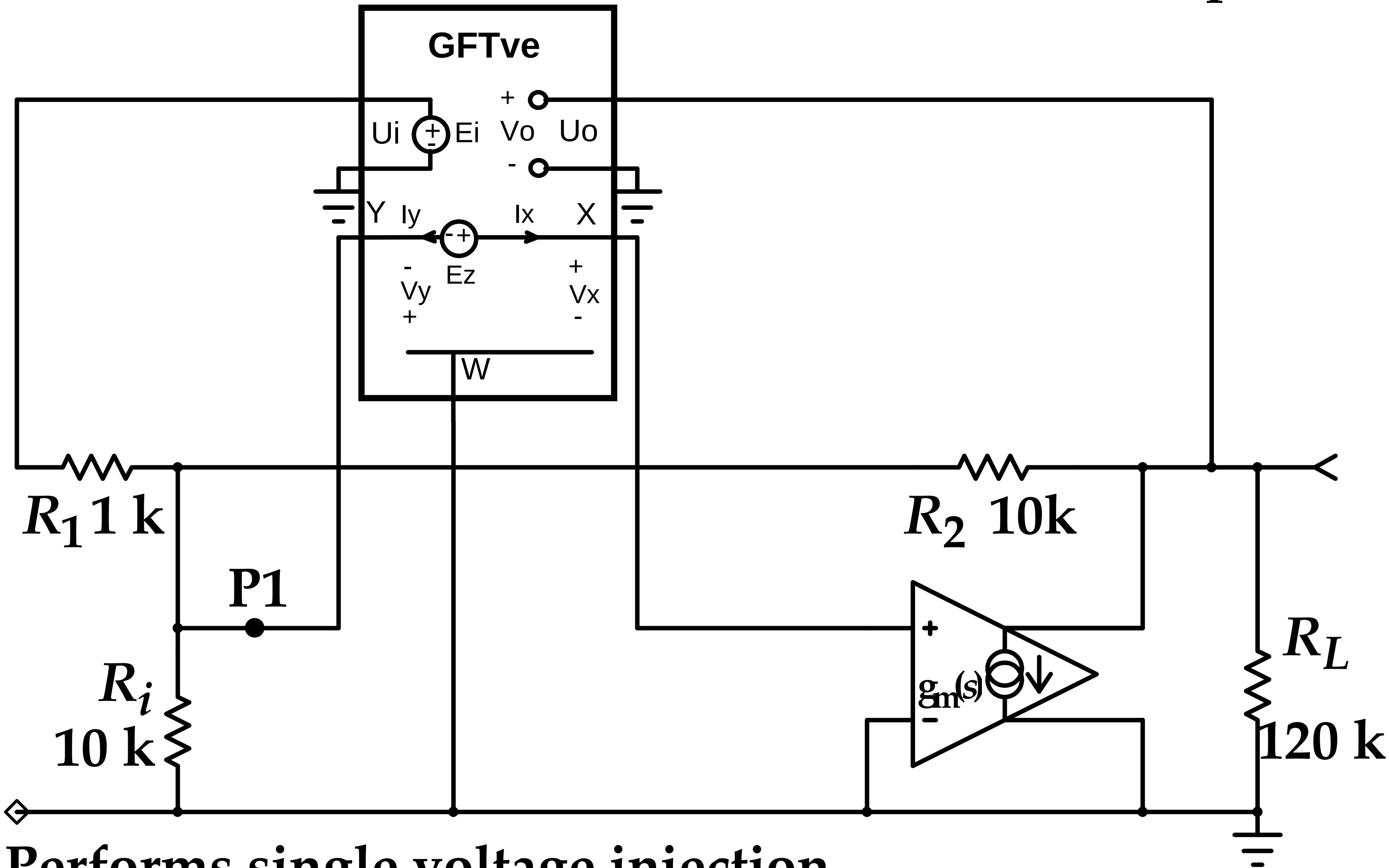
First version:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} H = H_\infty D + H_0 D_0$$

Second version:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} \qquad = H_\infty D D_n$$

Intusoft ICAP/4 Circuit Simulator with GFT Template



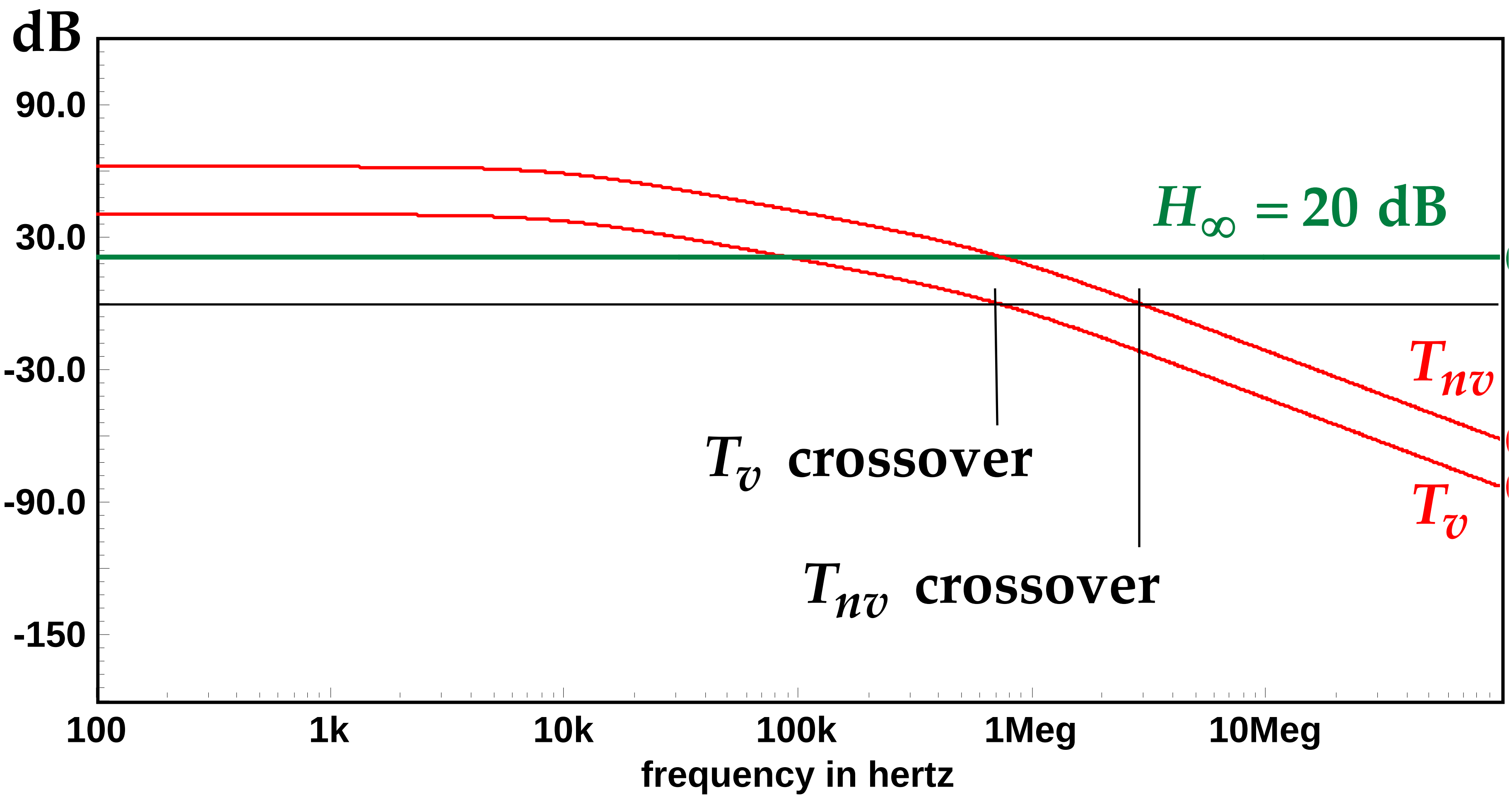
Performs single voltage injection at the ideal point P1

The GFT Template calculates all the second-level TFs H_∞, T, H_0, T_n and inserts them into any version of the first-level TF $H,$ for comparison with the directly calculated H (the "normal" closed-loop gain with no injected test signal).

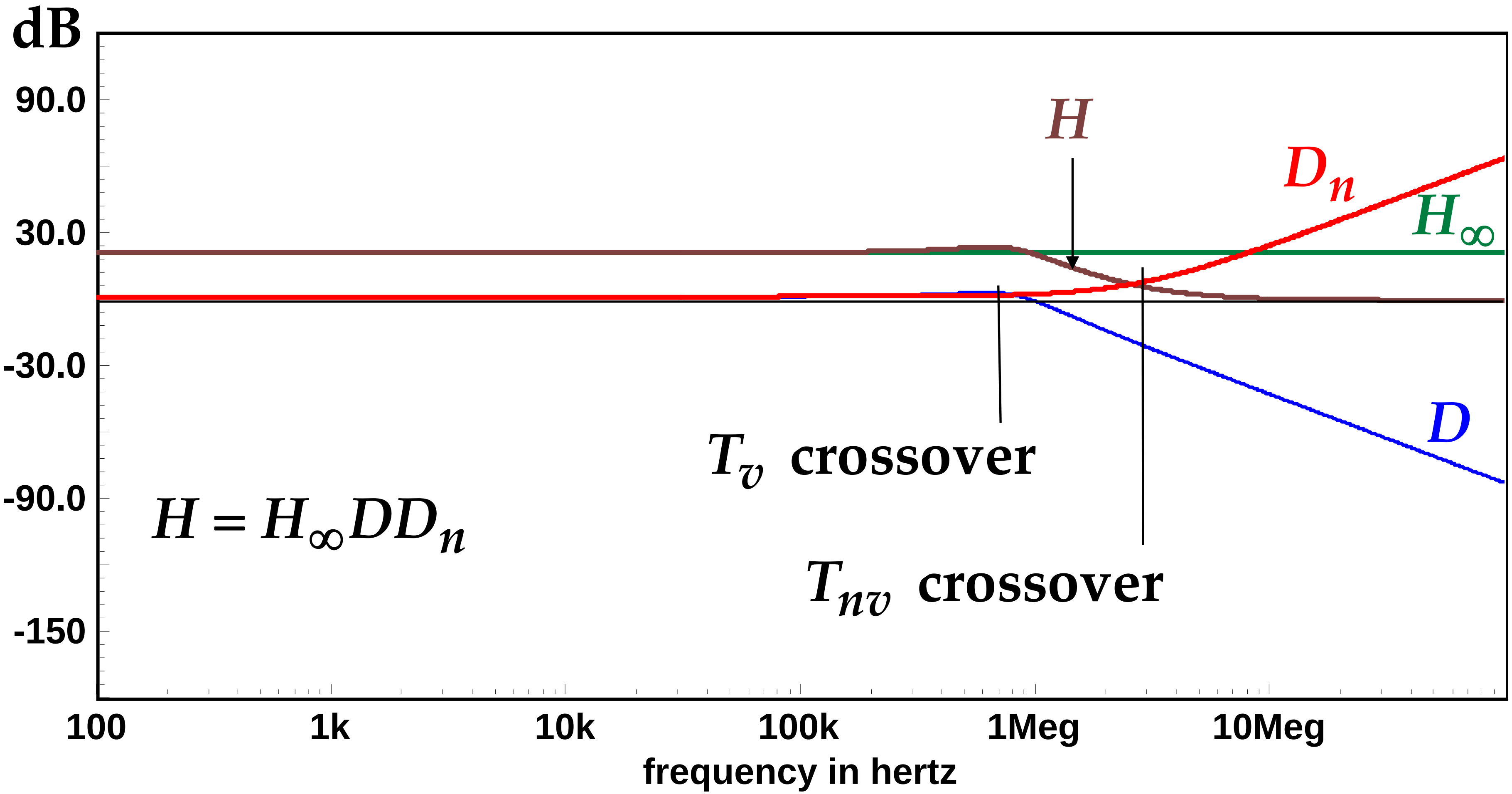
All TFs can be displayed as magnitude and phase Bode plots.

The results for the above circuit were those used previously to illustrate the four Feedback Amplifier Design Steps.

Check that H is what was expected



Check that H[calculated] is same as H[direct simulation]



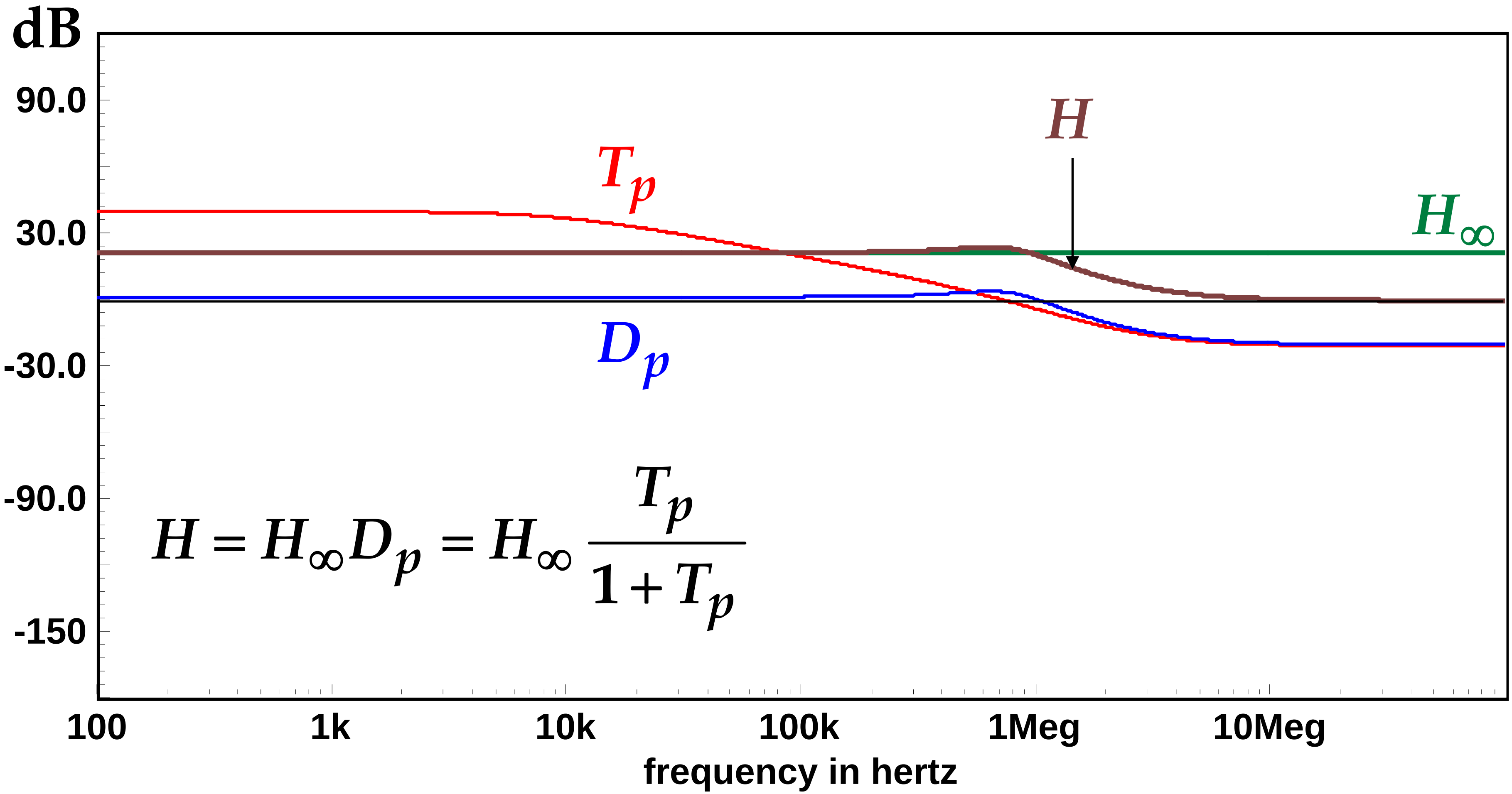
A third version of H can be found by forcing the result to be of the form $H = H_\infty \frac{1}{1+1/T_p}$:

$$H = H_\infty D D_n = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T_n}+\frac{1}{T}-\frac{1}{T_n}}$$

$$= H_\infty \frac{1}{1+\frac{T_n}{1+T_n}\left(\frac{1}{T}-\frac{1}{T_n}\right)} = H_\infty \frac{1}{1+\frac{1}{T_p}} = H_\infty D_p$$

where $T_p \equiv \frac{D_n}{\frac{1}{T}-\frac{1}{T_n}}$ **is a "pseudo loop gain"**

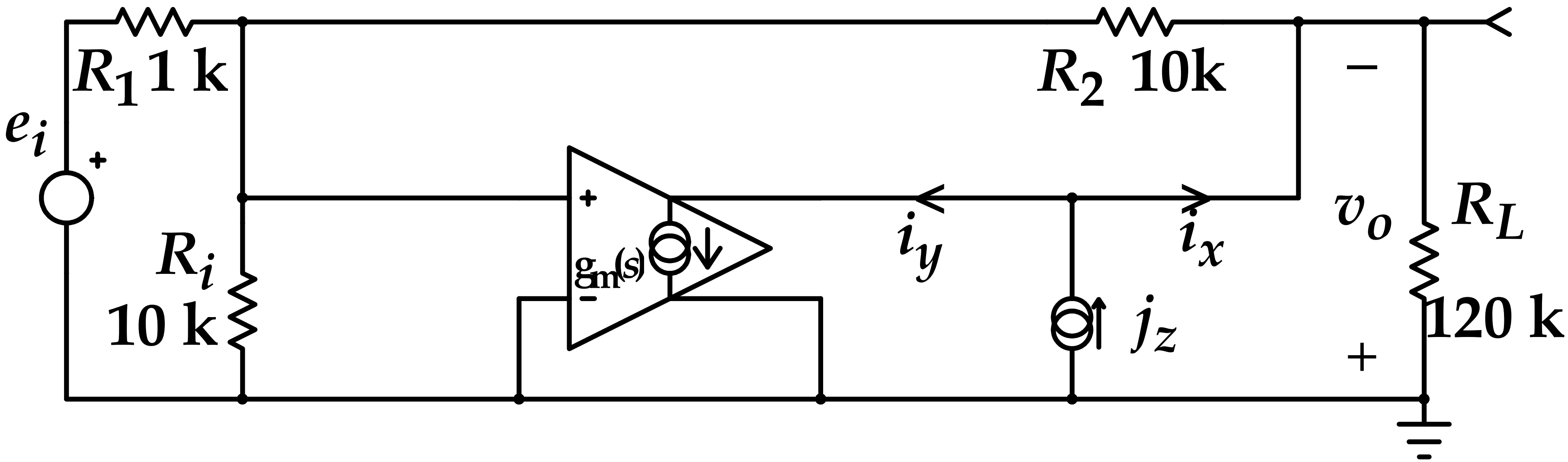
and $D_p \equiv \frac{T_p}{1+T_p}$ **is a "pseudo discrepancy factor"**



$$H = H_\infty D_p = H_\infty \frac{T_p}{1+T_p}$$

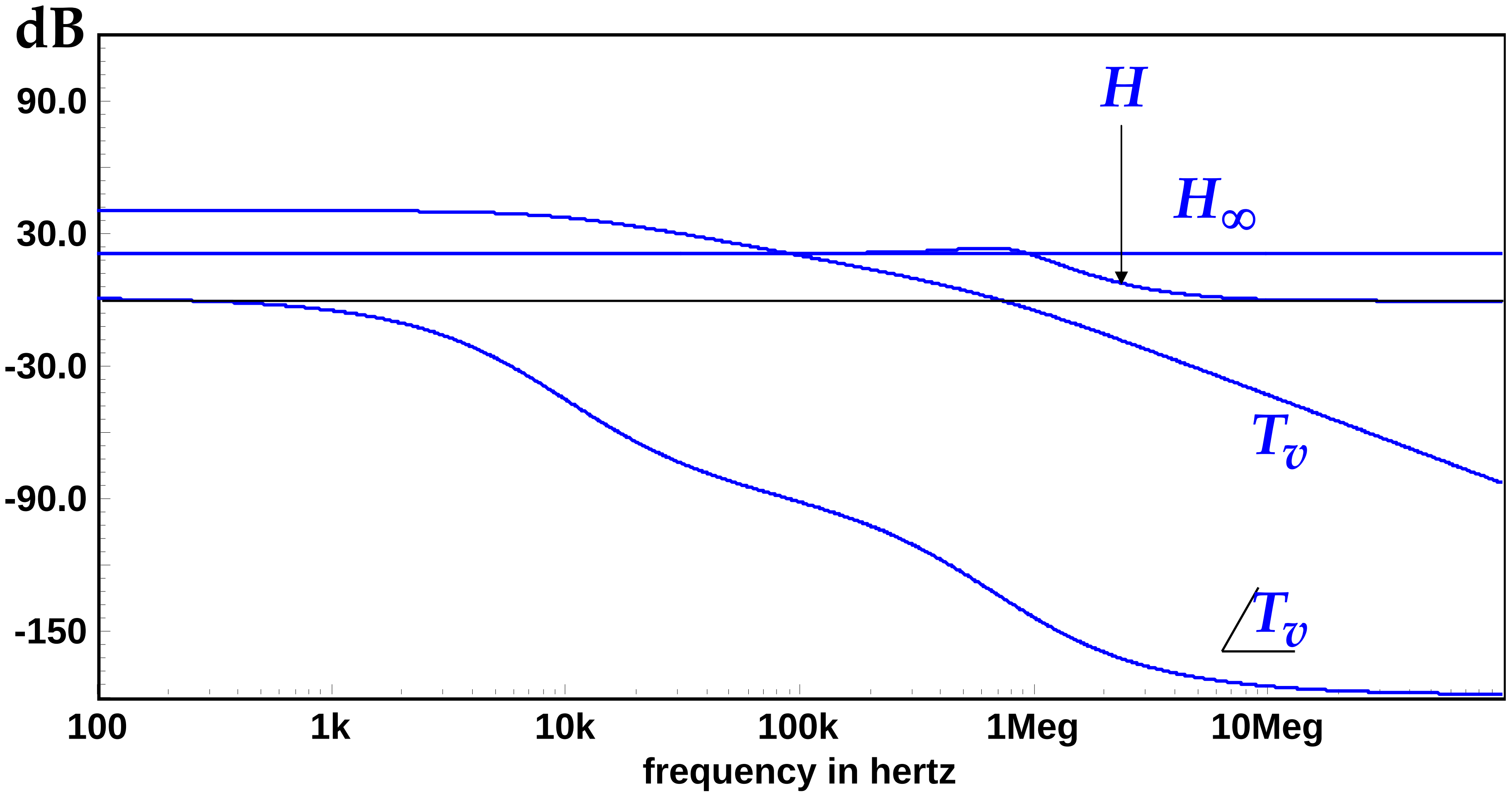
The pseudo loop gain T_p may be hard to relate to the circuit model, because it contains both T and T_n

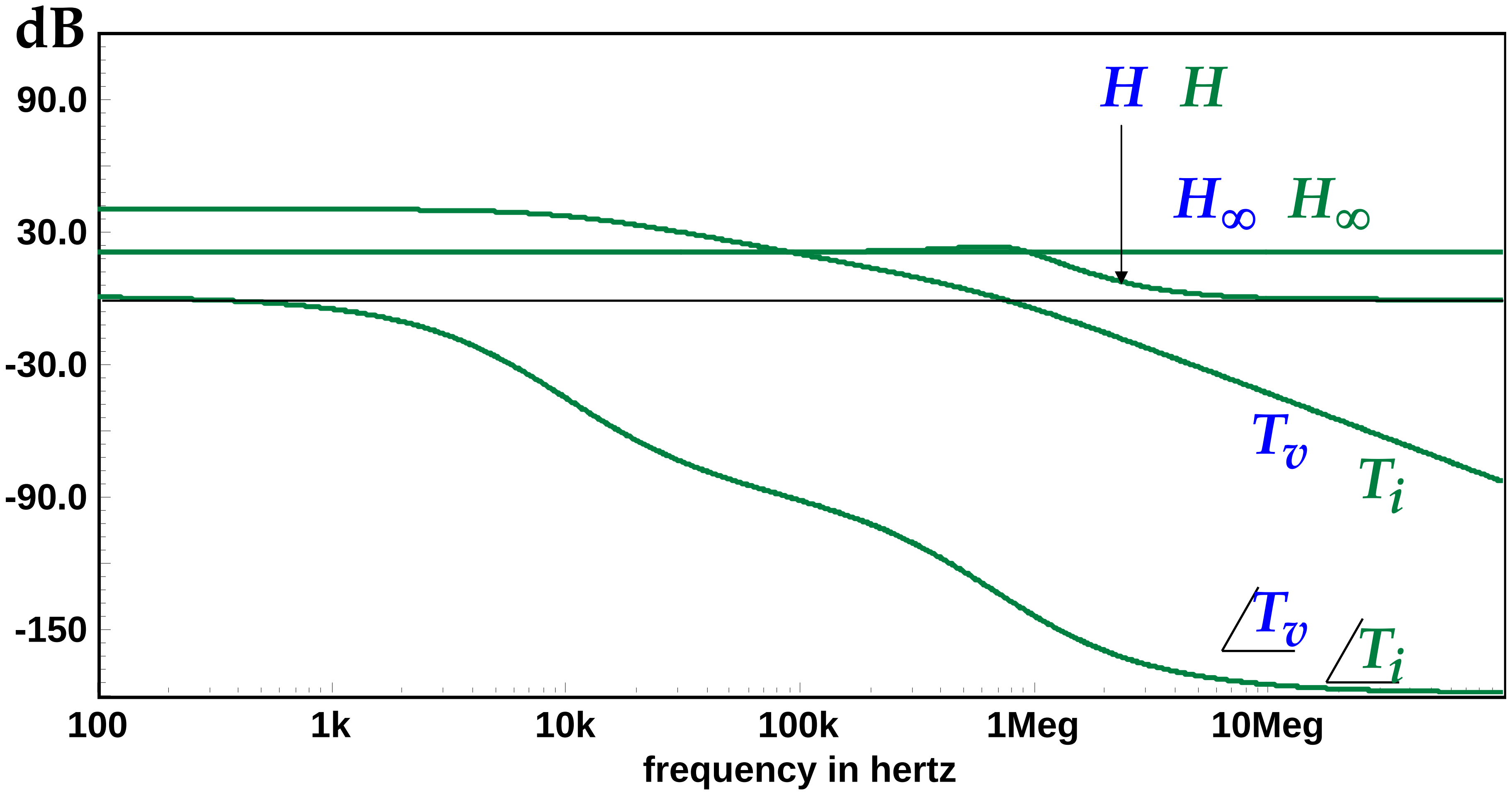
Ideal current injection point



An ideal current injection point is where i_y comes from an ideal (infinite impedance) current generator, *or* where i_x looks into a zero impedance.

Results are exactly the same as for the ideal voltage injection point.


dB
90.0
30.0
-30.0
-90.0
-150
H
H∞
Tv
Tv
100
1k
10k
100k
1Meg
10Meg
frequency in hertz


dB
90.0
30.0
-30.0
-90.0
-150
H
H
H∞
H∞
Tv
Ti
Tv
Ti
100
1k
10k
100k
1Meg
10Meg
frequency in hertz